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ABSTRACT

North Carolina revised its "Standard Course of Study" in 1985 to reflect the knowledge, skills, and attitudes needed to function in an industrial age. The entrance into what is being called the information age brings new challenges in preparing students with new skills in the areas of creative thinking, problem solving, interpersonal relations, communication, and cooperative learning. This handbook is a revision of the 1985 edition of the Teacher's Handbook for the Mathematics 9-12 Curriculum of the Standard Course of Study. Opening sections include the educational philosophy for the mathematics program, an outline of the mathematical concepts and student attitudes relative to the mathematics curriculum, the major emphases of the mathematics curriculum, expected program outcomes, and a diagram that traces the possible pathways through the mathematics program. The remaining two sections of the document describe the competency goals and objectives for the courses offered in the Standard Course of Study and supply sample measures corresponding to those competency goals and objectives. The courses in the Standard Course of Study are as follows: (1) Pre-Algebra; (2) Algebra I-A; (3) Algebra I-B; (4) Algebra I; (5) Technical Mathematics; (6) Geometry; (7) Algebra II; (8) Advanced Mathematics; and (9) Advanced Placement Calculus. (MDH)

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NORTH CAROLINA COMPETENCY-BASED CURRICULUM

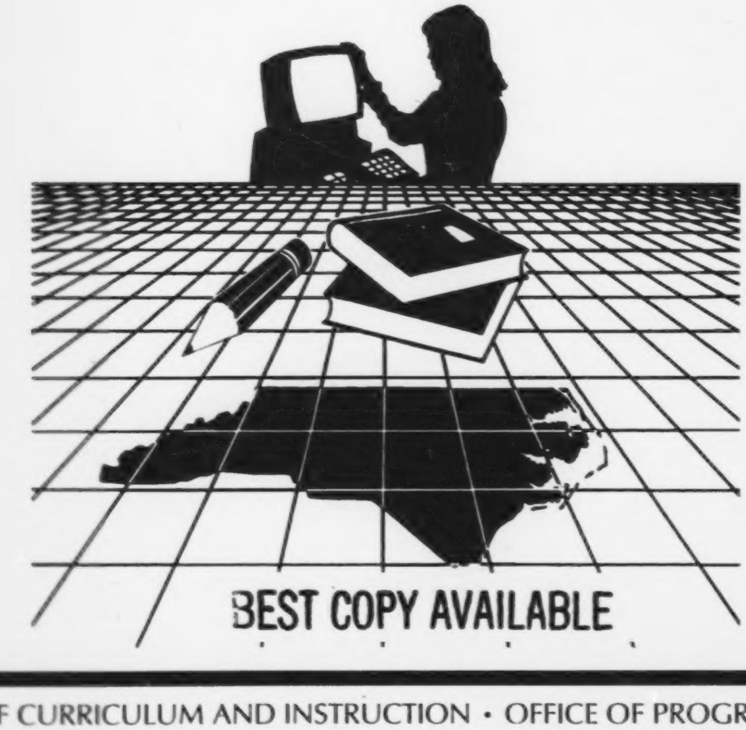
· GRADE-BY-GRADE ·

TEACHER HANDBOOK

Mathematics

9-12

ED 372 910



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DIVISION OF CURRICULUM AND INSTRUCTION · OFFICE OF PROGRAM SERVICES
NORTH CAROLINA DEPARTMENT OF PUBLIC INSTRUCTION

NORTH CAROLINA COMPETENCY-BASED CURRICULUM

· GRADE-BY-GRADE ·

TEACHER HANDBOOK

Mathematics

9-12

DIVISION OF CURRICULUM AND INSTRUCTION · OFFICE OF PROGRAM SERVICES
NORTH CAROLINA DEPARTMENT OF PUBLIC INSTRUCTION · 1992

ATTENTION!

North Carolina's ***Standard Course of Study*** and ***Teacher Handbooks*** for a competency-based curriculum are currently being revised. This booklet represents one in a series of revisions that will be shipped to replace the 1985 curriculum. These shipments will take place over the next three years. With each shipment, instructions will also be included on how to substitute the revised curriculum in place of existing pages within the Teacher Handbook manual.

REPLACEMENT INSTRUCTIONS for Mathematics 9 - 12 Curriculum

- REMOVE ALL PAGES FROM THE MATHEMATICS 9 - 12 TEACHER HANDBOOK.
- AFTER REMOVING DESIGNATED PAGES, REPLACE THOSE PAGES WITH ALL PAGES FROM THIS PACKET.
- IF YOU HAVE QUESTIONS, PLEASE CONTACT WANDRA POLK, 919-733-3512.

FOREWORD

North Carolina has had a ***Standard Course of Study*** since 1898. Since that time, the curriculum has periodically been revised to reflect the changing needs of students and society. The 1985 ***Standard Course of Study***, for example, reflected the knowledge, skills, and attitudes needed to function effectively in an industrial age. Moreover, it also included initial efforts to develop mature thinkers and problem solvers.

In the 1980's and early 90's we have witnessed a dramatic shift in the needs of business and industry and of society in general. These changes have been collectively heralded as the information age. The 21st century will bring new challenges in preparing students for the demands of an information age. While students continue to need mastery of such enabling skills as reading, writing, and computing, they must also master the new basics which include creative thinking/problem solving, interpersonal skills, negotiation and teamwork.

These changes, coupled with more in-depth learning at a much higher level, provide the foundation for current revisions to the ***Standard Course of Study*** and ***Teacher Handbook.*** The revisions are futuristic in outlook. They look at what students will need to know, be able to do, and be like. In other words, the curriculum focuses on the desired outcomes for North Carolina's students after thirteen years of schooling.

The North Carolina General Assembly is continuing its commitment to the development of a basic education program. This program includes the staffing and material support needed for the full implementation of the ***North Carolina Standard Course of Study*** and the revised curriculum in all public schools throughout the State. The financial support of the General Assembly and the work of educators throughout the State in developing the revised curriculum are essential contributions to the continuing efforts to provide a quality education for each North Carolina child.

Bob Etheridge

Bob Etheridge
State Superintendent of Public Instruction

ACKNOWLEDGEMENTS

The Department of Public Instruction gratefully acknowledges the cooperation and assistance received from individuals and groups throughout the State in this current revision process. Without such cooperation, the revisions and printing of the ***North Carolina Standard Course of Study*** and the ***Teacher Handbook*** would not have been possible.

We wish to express a special thanks to:

- the Division of Curriculum and Instruction for providing the leadership and vision that guided the development of these documents. The untiring efforts of this staff, directed by Joseph B. Webb, contributed greatly to the completion of this task,
- the many local educators who participated in the current revision process by serving on curriculum committees and reacting to draft documents,
- faculty from the institutions of higher education who advised the staff and assisted in the revision of the curriculum,
- Raleigh-based staff in Arts Education, Communication Skills, Computer Services, Exceptional Children, Healthful Living, School Media Programs, Mathematics, Second Language Studies, Science, Social Studies, and Vocational Education. These Public Instruction staff members carried the primary responsibility for revising and editing the curriculum,
- office support staff in each of the above named areas who, in addition to their on-going responsibilities, word processed the revised documents,
- Technical Assistance staff who reacted to revised drafts,
- the Division of Communication Services for technical assistance in the publication of the documents,
- Association for Supervision and Curriculum Development (ASCD) for allowing its ***Dimensions of Thinking*** to serve as the framework for this revision process,
- members of the Agency's Thinking Skills Task Force who worked tirelessly to craft the thinking skills philosophy that is expressed in the Introduction,
- especially Wandra Polk who coordinated the revision of the ***Standard Course of Study*** and ***Teacher Handbook***.

The current revision process, involved on some level, the entire education community, and its end product is a North Carolina curriculum of which the State can be justifiably proud. We will constantly revise and improve the ***Competency-Based Curriculum*** in order that it will continue to meet the needs of the children of North Carolina.

Table of Contents

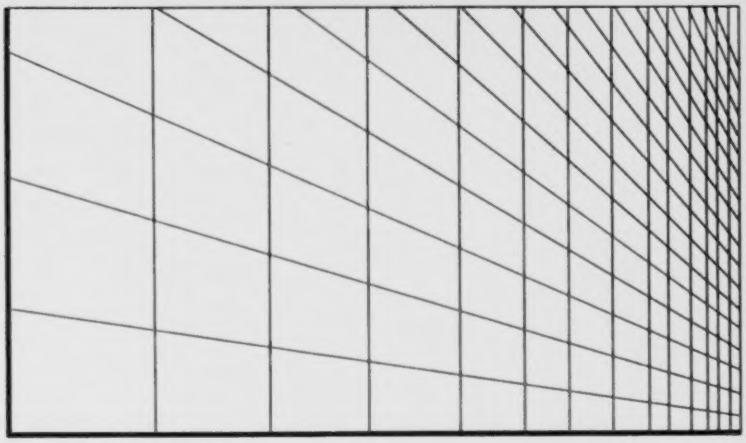
Foreword iii
Acknowledgements iv
Philosophy 3
Mathematics Curriculum 4
Major Emphases 6
Program Outcomes 7
Pathways through the Mathematics Program 8

Competency Goals and Objectives

- Pre-Algebra 9
- Algebra I - A 12
- Algebra I - B 15
- Algebra I 19
- Technical Mathematics 22
- Geometry 25
- Algebra II 28
- Advanced Mathematics 31
- Advanced Placement Calculus 34

Competency Goals and Objectives with Sample Measures

- Pre-Algebra 39
- Algebra I - A 79
- Algebra I - B 107
- Algebra I 147
- Technical Mathematics 189
- Geometry 229
- Algebra II 277
- Advanced Mathematics 323
- Advanced Placement Calculus 361



Mathematics

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MATHEMATICS

PHILOSOPHY

The primary goal of mathematics education is to ensure that every student will become mathematically literate. Literacy implies that a student (1) is a problem solver; (2) is confident in his/her ability to do mathematics; (3) is able to communicate mathematically; (4) is able to reason mathematically; and (5) values mathematics. Mathematical literacy is a necessity for the layperson as well as for the professional who chooses a career in mathematics or in a field which makes use of mathematics.

Cultural changes occurring in our time have significant implications for the curriculum as a whole and for mathematics in particular. The technological world in which we live makes it imperative that every citizen have some understanding of mathematical method.

The mathematics program proposed here is by necessity broader and more inclusive than in the past. It must develop more than vocabulary, facts, and principles; more than the ability to analyze a problem situation; more than an understanding of the logical structure of mathematics. The mathematics program must provide students with the knowledge which will enable them to distinguish fact from opinion, relevant from irrelevant data, and experimental results from proven theorems. This program has to stimulate curiosity so that students will enjoy exploring new ideas and creating mathematics which is new for them even though it has been discovered by others. It must develop reading skills, motivation, and study habits essential for independent learning of mathematics. It must develop in students the appreciation for the connections between various branches of mathematics and how mathematics is connected to other disciplines. In short, the mathematics program must produce students who know how to learn mathematics, enjoy learning mathematics, and are motivated to continue their learning. All of the mathematics courses have the goal of developing both competence and confidence in mathematics for all students.

Computers and calculators should be integrated into the mathematics curriculum in imaginative ways. In addition to their use with numerical calculations, they should be used to graph equations, display data, clarify concepts and computational algorithms. They should be used extensively in problem solving situations as they enable a student to attack a problem from different points of view.

MATHEMATICS CURRICULUM

I. Mathematical Concepts and Skills

At each grade level or in each course outlined, mathematical concepts and skills are continuously developed and reinforced.

K–5
Elementary Mathematics

6 – 8
Middle School/Junior High Mathematics

9 – 12

Pre-Algebra
Algebra I – A
Algebra I – B
Technical Mathematics

Algebra I
Geometry
Algebra II
Advanced Mathematics
AP Calculus

(Other Electives)

II. Skills and Attitudes

In addition to the formal mathematical concepts and skills that students acquire on a continuing basis, there are other benefits derived from the study of mathematics. Students develop attitudes relative to mathematics and its importance in their lives; and they acquire skills which enable them to process information, analyze data, and draw conclusions essential to sound decision-making.

A. Positive Attitudes Concerning:
   1. The contributions of mathematics, science, and technology in shaping the world in which people live—adverse as well as beneficial effects.
   2. The role of mathematics in helping people meet their responsibilities to society.
   3. The learning and experiencing of mathematics.
   4. The use of scientific inquiry as a way of thinking and evaluating all human activity.
   5. The historical background of mathematics.

B. Process Skills
   1. Observing
   2. Classifying
   3. Using space/time relations
   4. Using numbers
   5. Communicating
   6. Measuring
   7. Inferring
   8. Predicting
   9. Interpreting data
   10. Formulating hypotheses
   11. Formulating models

MAJOR EMPHASES

The aim of the high school mathematics curriculum is to provide every student with appropriate mathematical content that is broad in scope. Some of the content should be immediately useful to students in their role as consumers and part-time employees. The content must also enable students to study higher level mathematics. The mathematics curriculum then should have the flexibility to help prepare students for many different careers and vocations.

For students having a high aptitude in mathematics, courses including Algebra I, Geometry, Algebra II, Advanced Mathematics, and Calculus are offered. An alternative program consists primarily of courses in Pre-Algebra, Algebra I (A and B), and Technical Mathematics. High schools must provide appropriate remediation to assist students in passing the Competency Test and to develop necessary skills to enter other mathematics courses. **Each course in the high school program should include the use of calculators and computers.**

A basic high school mathematics program should contain the following course offerings:

Pre-Algebra
Algebra I - A
Algebra I - B
Technical Mathematics

Algebra I
Geometry
Algebra II
Advanced Mathematics
AP Calculus

Other Electives

Other electives may be offered depending upon the school's resources and size.

Shown in Figure I is a framework for planning secondary school mathematics programs entitled, Pathways through the Mathematics Program. This framework provides all students access to four years of quality mathematics courses which will prepare them for the world of work or higher education. It enables students to alter their path through the program depending upon the degree of success they experience with certain courses. The grade level under which a course is listed indicates where it is ordinarily taught. However, this listing should be used only as a suggestion. With careful planning, any course listed might be taught at a different grade level.

PROGRAM OUTCOMES

A balanced and effective mathematics program, K-12, prepares students to be mathematically literate. Literacy implies that a student: (1) is a good critical thinker and problem solver; (2) is able to communicate mathematically; (3) is able to reason mathematically; (4) values mathematics; and (5) is confident in his/her ability to do mathematics. Students gain from the mathematics program the knowledge, skills, and attitudes which enable them to be productive citizens in a world that demands of its people the ability to communicate, to solve problems, and use technology.

Students successfully completing a balanced and effective mathematics program:

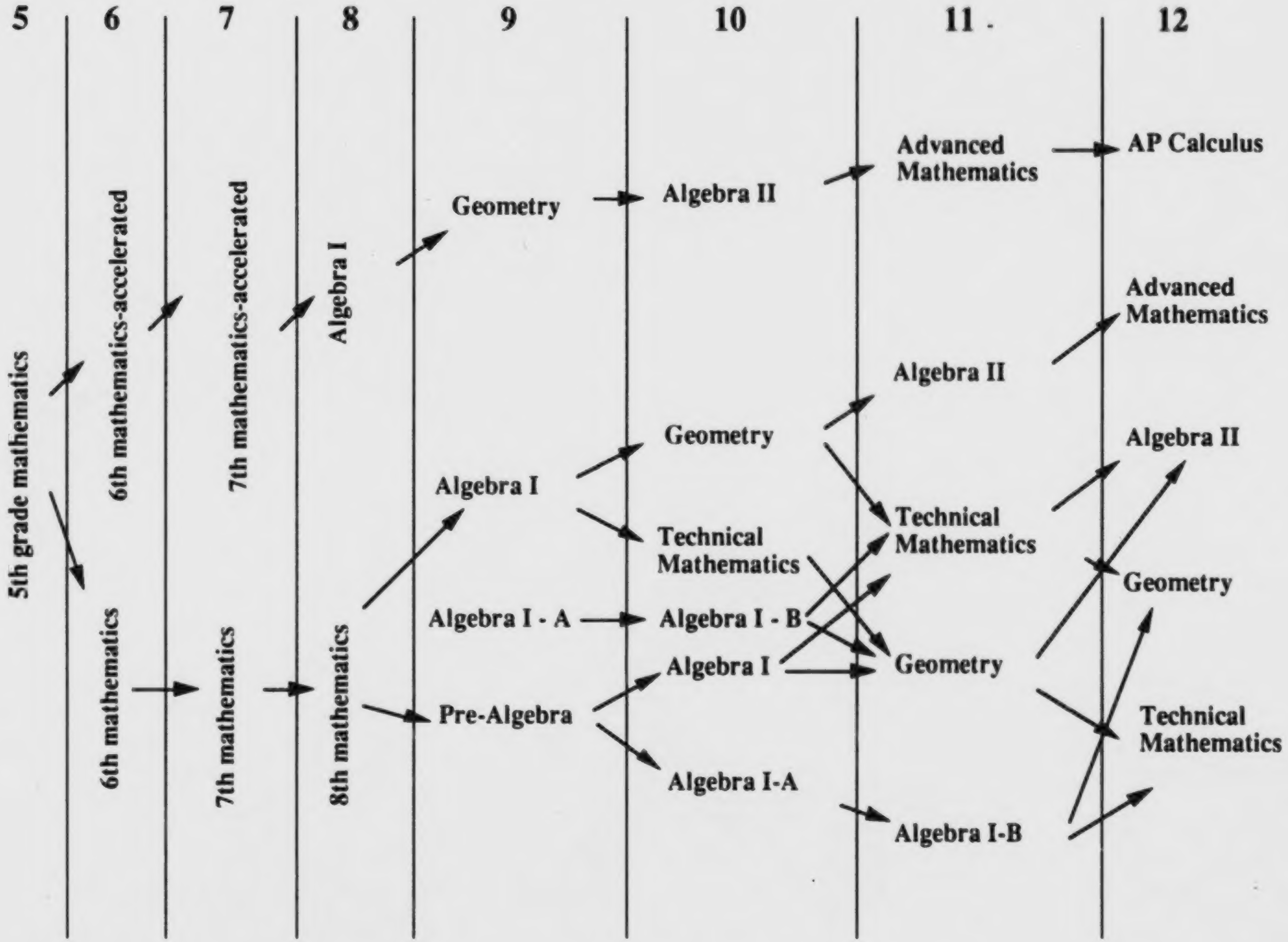
- are able to use critical thinking to solve problems.
- are able to communicate their mathematical ideas to others.
- display confidence in themselves as mathematical problem solvers.
- value mathematics for its structure and for its usefulness.
- use technology skillfully to analyze and solve problems.

The mathematics outcomes-based curriculum:

- is designed for statewide use.
- is designed to provide a common core of quality mathematics to all students.
- is not correlated with any textbook and does not restrict the use of any relevant textbook materials, or program.
- encourages the offering of a series of courses suitable to varying abilities of all students. For example, many schools offer Algebra I below grade 9.
- assumes that there are individual differences among students and that the degree and rate of achievement will vary among them.
- assumes that teachers are different and will approach the teaching of mathematics differently.
- stresses a balance of concept learning, computational processes, and problem solving.
- encourages extensive use of technology to enhance, enrich, and extend the learning experiences in mathematics for all students.

Figure 1

Pathways through the Mathematics Program


5
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11
12
5th grade mathematics
6th mathematics-accelerated
7th mathematics-accelerated
Algebra I
6th mathematics
7th mathematics
8th mathematics
Geometry
Algebra I
Algebra I - A
Pre-Algebra
Algebra II
Geometry
Technical Mathematics
Algebra I - B
Algebra I
Algebra I-A
Advanced Mathematics
Algebra II
Technical Mathematics
Geometry
Algebra I-B
AP Calculus
Advanced Mathematics
Algebra II
Geometry
Technical Mathematics

Pre-Algebra

COMPETENCY GOAL 1: The learner will demonstrate an understanding of and use real numbers to solve problems.

1.1 Read, write and use numbers in various forms, including fractions, decimals, percents, and exponential notation.
1.2 Write numbers in scientific notation, using a calculator when appropriate.
1.3 Use patterns to explore the rules of divisibility.
1.4 Use exponential notation to express the prime factorization of a number.
1.5 Find the greatest common factor (GCF) and the least common multiple (LCM) of two or more numbers.
1.6 Compare two or more rational numbers.
1.7 Estimate sums, differences, products, and quotients of real numbers.
1.8 Use a table of squares or a calculator to find the square root of a number.
1.9 Estimate the square root of a number.
1.10 Demonstrate an understanding of the relationship between fractions, decimals, percents, and ratios.
1.11 Relate common fractions to frequently used percents; estimate and calculate using these percents.
1.12 Find the missing term of a proportion.
1.13 Find the percent of a given number.
1.14 Find what percent one number is of another.
1.15 Find a number when a percent of it is given.
1.16 Write fractions as equivalent, terminating, or repeating decimals.
1.17 Solve problems involving discount, markup, commission, profit, simple and compound interest.
1.18 Write terminating or repeating decimals as equivalent fractions.
1.19 Find the absolute value of a real number.
1.20 Use laws of exponents to write expressions in equivalent forms.

COMPETENCY GOAL 2: The learner will demonstrate an understanding of and use properties and relationships of geometry.

2.1 Classify angles (interior, exterior, complementary, supplementary) and pairs of lines, including skew lines, and solve problems involving them.
2.2 Identify and distinguish among similar, congruent, and symmetric figures; name corresponding parts.
2.3 Find the lengths of the sides of similar figures.
2.4 Use the Pythagorean Theorem to find the missing side of a right triangle using a table of square roots, calculator, or mental math.
2.5 Find the lengths of the missing sides of 30° -60° -90° and 45° -45° -90° right triangles given the length of one side.
2.6 Graph similar geometric figures on a coordinate plane.
2.7 Solve problems that relate geometric concepts to real-world situations.

COMPETENCY GOAL 3: The learner will demonstrate an understanding of pre-algebra.

3.1 Use order of operations to simplify numerical expressions, verifying the results with a calculator or computer.
3.2 Identify the commutative, associative and distributive properties.
3.3 Evaluate algebraic expressions.
3.4 Solve simple equations algebraically.
3.5 Solve simple inequalities and graph the solutions.
3.6 Use graphs and tables to represent relations and functions of ordered pairs.
3.7 Describe, extend, analyze and create a variety of geometric and numerical patterns to investigate relationships and solve problems.
3.8 Given an equation, formulate a problem.
3.9 Write an equation which could be used to solve a given problem.
3.10 Given a rule or function that describes a linear equation, generate ordered pairs and graph the equation.
3.11 Given a formula make appropriate substitutions and solve for one unknown.

COMPETENCY GOAL 4: The learner will demonstrate an understanding and use of measurement.

4.1 Apply measurement concepts and skills involving area, perimeter, and circumference in problem-solving situations.
4.2 Determine the effect on area and perimeter when changing one or two of the dimensions of a rectangle.
4.3 Find the surface area of rectangular solids, pyramids, cones and cylinders.
4.4 Find the volume of prisms, pyramids, cylinders and cones.
4.5 Estimate solutions and solve problems related to volumes of rectangular solids.

COMPETENCY GOAL 5: The learner will solve problems and reason mathematically.

5.1 Represent situations verbally, numerically, graphically, geometrically, or symbolically and use a variety of strategies to solve non-routine problems.
5.2 Discuss alternate strategies, evaluate outcomes, and make conjectures and generalizations based on problem situations.
5.3 Use concrete or pictoral models to solve problems.
5.4 Solve problems involving the interpretation of graphs including inferences and conjectures.
5.5 Make and evaluate conjectures and arguments, using deductive and inductive reasoning.
5.6 Investigate open-ended problems, formulate questions, and extend problem-solving situations.
5.7 Use proportional reasoning to solve problems.

COMPETENCY GOAL 6: The learner will demonstrate an understanding and use of probability and statistics.

6.1 Create, compare, and evaluate different graphic representations of the same data.
6.2 Use measures of central tendency (mean, mode, and median) and range to describe data; compare two sets of unequal data.
6.3 Evaluate approximate uses of different measures of central tendency in problem-solving situations.
6.4 Draw inferences and construct convincing arguments based on analysis of data.
6.5 Investigate and recognize misuses of statistical or numerical data.
6.6 Find the mathematical and experimental probability of simple and compound events using experiments, random number generation, computer simulation, and theoretical methods.
6.7 Collect data involving two variables and display them on a scatter plot.
6.8 Interpret data displayed on a scatter plot.
6.9 Use mathematical probability and experimental results for making predictions and decisions.

COMPETENCY GOAL 7: The learner will use fractions, decimals, percents, and integers to solve problems.

7.1 Use operations with whole numbers, decimals, fractions, and integers to solve problems.
7.2 Select appropriate operations and strategies for solving a variety of appliedproblems involving fractions, decimals, and percents and justify the selections using calculators, mental math, and estimates.
7.3 Estimate the answer to and solve problems using ratio, proportion, and percent.
7.4 Use ratio, proportion, and percent to solve real-world problems.

Algebra I - A

COMPETENCY GOAL 1: The learner will use the language of algebra.

1.1 Simplify numerical expressions.
1.2 Use "order of operations" to simplify numerical expressions.
1.3 Use grouping symbols to indicate order of operations.
1.4 Evaluate variable expressions.
1.5 Simplify exponential expressions.
1.6 Write expressions in exponential form.
1.7 Translate word phrases into variable expressions.
1.8 Translate variable expressions and equations into word phrases.

COMPETENCY GOAL 2: The learner will identify and apply properties of real numbers.

2.1 Identify and apply the properties of zero for both addition and multiplication.
2.2 Identify and apply the property of one for multiplication.
2.3 Identify and apply the commutative and associative properties of addition and multiplication to simplify expressions or computational processes.
2.4 Identify and apply the distributive property.

COMPETENCY GOAL 3: The learner will solve linear equations and inequalities in one variable.

3.1 Solve an equation by using the addition or subtraction property of equality.
3.2 Solve an equation by using the multiplication or division property of equality.
3.3 Solve an equation by using more than one property of equality.
3.4 Solve an equation graphically.
3.5 Solve an equation by using the distributive property.
3.6 Solve an equation which contains similar terms.
3.7 Find the solution set for a linear inequality when replacement values are given for the variable.
3.8 Solve a linear inequality by using transformations.
3.9 Use a linear equation to solve problems.
3.10 Use formulas to solve problems.
3.11 Use inequalities to solve problems.

COMPETENCY GOAL 4: The learner will use ratios, proportions, and percents to solve problems.

4.1 Write a ratio in lowest terms.
4.2 Solve a proportion.
4.3 Use a ratio or a proportion to solve problems.
4.4 Solve problems involving percents.

COMPETENCY GOAL 5: The learner will perform operations with real numbers.

5.1 Determine the additive or multiplicative inverse of a number.
5.2 Distinguish between rational and irrational numbers.
5.3 Determine the absolute value of expressions.
5.4 Compare real number expressions.
5.5 Simplify variable expressions involving addition and subtraction.
5.6 Simplify variable expressions involving multiplication and division.
5.7 Simplify real number expressions with or without a calculator.
5.8 Simplify radicals.
5.9 Multiply or divide two radical expressions.
5.10 Use a calculator to find the approximate square root of a number.
5.11 Add or subtract radical expressions.
5.12 Use the Pythagorean Theorem to solve problems.

COMPETENCY GOAL 6: The learner will demonstrate an elementary understanding of relations and functions.

6.1 Graph and locate sets of real numbers on a number line.
6.2 Graph ordered pairs of numbers on the coordinate plane and interpret information related to these sets of points.
6.3 Find the distance between two points on a number line.
6.4 Graph a relation on the coordinate plane.
6.5 Graph a relation given an equation and a domain.
6.6 Identify intercepts as numbers.

COMPETENCY GOAL 7: The learner will solve and graph linear equations and inequalities.

7.1 Graph an inequality on a number line.
7.2 Solve an inequality and graph its solution set.
7.3 Determine if data are behaving in a linear fashion.
7.4 Find the solution set of open sentences in two variables when given replacement sets for the variables.
7.5 Graph a linear equation in two variables.

COMPETENCY GOAL 8: The learner will solve linear equations with two variables.

8.1 Solve systems of linear equations.
8.2 Use substitution to solve systems of linear equations.
8.3 Identify inconsistent systems.
8.4 Use a computer or graphics calculator to solve systems of linear equations.

COMPETENCY GOAL 9: The learner will perform operations with polynomials.

9.1 Use models to represent polynomials.
9.2 Use models to find the sum or difference of two polynomials.
9.3 Add and subtract polynomials.
9.4 Multiply monomials.
9.5 Find an indicated power of a monomial.
9.6 Multiply a polynomial by a monomial.
9.7 Divide two monomials.

COMPETENCY GOAL 10: The learner will simplify expressions with algebraic fractions.

10.1 Simplify algebraic fractions.
10.2 Use the distributive property to simplify fractions.
10.3 Multiply algebraic fractions.
10.4 Divide algebraic fractions.

Algebra I - B

COMPETENCY GOAL 1: The learner will perform operations with real numbers.

1.1 Add or subtract real numbers.
1.2 Multiply or divide real numbers.
1.3 Use the order of operations to simplify expressions.
1.4 Evaluate exponential expressions.
1.5 Evaluate polynomials.
1.6 Interpret zero as an exponent.
1.7 Interpret negative exponents.
1.8 Write numbers in scientific notation and use this notation with the calculator.
1.9 Simplify real number expressions with and without a calculator.

COMPETENCY GOAL 2: The learner will identify the properties of real numbers and apply them to numerical and algebraic expressions.

2.1 Recognize and use properties of addition.
2.2 Recognize and use properties of multiplication.
2.3 Use the distributive property to simplify expressions.
2.4 Use the distributive property to factor a common monomial term in an algebraic expression.
2.5 Use number properties to combine like terms.

COMPETENCY GOAL 3: The learner will solve linear equations and inequalities with one variable and graph their solutions.

3.1 Solve an equation by using the addition property of equality and the idea of an additive inverse.
3.2 Solve an equation by using the multiplication property of equality and the idea of a multiplicative inverse.
3.3 Solve an equation graphically or by using more than one property of equality.
3.4 Solve an equation which has a variable in both members.
3.5 Translate word statements into equations and solve them.
3.6 Solve an equation in which the numerical coefficient is a fraction.
3.7 Solve a formula for one of its variables or find the value of a variable when values of other variables are given.
3.8 Use formulas to solve problems.
3.9 Graph an inequality on a number line.
3.10 Find the solution set of a linear inequality when replacement values are given for the variable.
3.11 Solve a linear inequality to solve problems.
3.12 Use inequalities to solve problems.
3.13 Find the solution set of combined inequalities.
3.14 Solve a simple equation involving absolute value.

COMPETENCY GOAL 4: The learner will demonstrate an elementary understanding of relations and functions.

4.1 Graph ordered pairs of numbers on the coordinate plane and interpret information related to those sets of points.
4.2 Graph a relation on the coordinate plane.
4.3 Distinguish between a relation and a function.
4.4 Graph a relation given an equation and a domain.
4.5 Sketch a reasonable graph for a given relationship.
4.6 Interpret a graph in a real-world setting.
4.7 Use a computer or graphing calculator to explore the graphs of functions.
4.8 Compare ordered pairs of numbers to the line $y = x$ and interpret the results.

COMPETENCY GOAL 5: The learner will simplify radical expressions which contain variables.

5.1 Simplify square root radicals.
5.2 Simplify radical expressions involving products and quotients.
5.3 Simplify the sums and differences of radical expressions.
5.4 Find the approximate square root of a number with and without a calculator.
5.5 Compare real number expressions.
5.6 Multiply two binomials which contain square roots.
5.7 Solve equations which contain radicals.

COMPETENCY GOAL 6: The learner will graph and solve systems of linear equations and inequalities.

6.1 Solve a system of two linear equations in two variables by graphing.
6.2 Solve a system of two linear equations by the substitution method.
6.3 Use the addition or subtraction method to find the solution to a pair of linear equations in two variables.
6.4 Use multiplication with the addition or subtraction method to solve a system of linear equations.
6.5 Use systems of linear equations to solve problems.

COMPETENCY GOAL 7: The learner will perform operations with polynomials.

7.1 Add and subtract polynomials.
7.2 Multiply polynomials.
7.3 Divide a polynomial by a monomial.
7.4 Divide polynomials.
7.5 Find the product of two binomials.
7.6 Factor a trinomial.
7.7 Find the square root of a binomial.
7.8 Factor a perfect square trinomial.
7.9 Find the product of a sum and a difference.
7.10 Factor the difference of two squares.

COMPETENCY GOAL 8: The learner will work with ratios, proportions, and percents.

8.1 Simplify ratios involving algebraic expressions.
8.2 Solve proportions.
8.3 Use ratios and proportions to solve problems.
8.4 Solve problems involving percents.

COMPETENCY GOAL 9: The learner will perform operations with algebraic fractions.

9.1 Find the Least Common Multiple (LCM) of algebraic expressions.
9.2 Add algebraic fractions with like denominators.
9.3 Subtract algebraic fractions with like denominators.
9.4 Add algebraic fractions with unlike denominators.
9.5 Subtract algebraic fractions with unlike denominators.

COMPETENCY GOAL 10: The learner will analyze linear equations.

10.1 Write linear equations in slope intercept form.
10.2 Find the slope of a non-vertical line given two points on the line.
10.3 Graph a line given its slope and *y*-intercept.
10.4 Determine the slope of a line from its graph.
10.5 Determine the slope of a line from its equation.

COMPETENCY GOAL 11: The learner will explore, graph, and interpret nonlinear equations.

11.1 Graph a quadratic equation.
11.2 Use an automatic grapher to find the solution to a quadratic equation.
11.3 Solve a quadratic equation when one member is in factored form and the other member is zero.
11.4 Solve a quadratic equation in which a perfect square equals a constant.
11.5 Solve a quadratic equation by factoring.
11.6 Understand that the vertex provides the maximum or minimum value of the function.
11.7 Solve a quadratic equation by using the quadratic formula.
11.8 Use quadratic equations to solve problems.
11.9 Determine if a set of data represents an exponential function.
11.10 Use formulas, calculators, and automatic graphers to explore and solveproblems involving exponentials.

Algebra I

COMPETENCY GOAL 1: The learner will use the language of algebra.

1.1 Evaluate algebraic expressions.
1.2 Use formulas to solve problems.
1.3 Translate word phrases and sentences into expressions and equations and vice versa.
1.4 Use the associative, commutative and distributive properties.

COMPETENCY GOAL 2: The learner will perform operations with real numbers.

2.1 Simplify real number expressions with and without a calculator.
2.2 Determine the additive or multiplicative inverse of a number.
2.3 Determine the absolute value of expressions.
2.4 Raise a real number to an indicated power.
2.5 Write numbers in scientific notation and use this notation with the calculator.
2.6 Distinguish between rational and irrational numbers.
2.7 Find approximations for square roots with and without a calcultor.
2.8 Simplify radical expressions.
2.9 Multiply two binomials which contain square roots.
2.10 Compare real number expressions.

COMPETENCY GOAL 3: The learner will solve equations and inequalities with one variable.

3.1 Solve a simple equation by using the addition property of equality and theidea of additive inverse.
3.2 Solve a simple equation by using the multiplication property of equality and the idea of multiplicative inverse.
3.3 Solve an equation graphically and by using more than one property of equality.
3.4 Solve an equation which contains similar terms.
3.5 Solve an equation which has the variable in both members.
3.6 Solve an equation in which the numerical coefficient is a fraction.
3.7 Solve a formula for one of its variables or find the value of a variable when values of the other variables are given.
3.8 Use problem solving skills to solve real-world and "word" problems which involve a linear equation or a formula.
3.9 Solve a simple equation involving absolute value.
3.10 Solve a simple equation containing a radical.
3.11 Find the solution set for a linear inequality when replacement values are given for the variables.
3.12 Solve a linear inequality by using transformations.
3.13 Use inequalities to solve problems.
3.14 Find the solution set of combined inequalities.

COMPETENCY GOAL 4: The learner will demonstrate an elementary understanding of relations and functions.

4.1 Graph and locate sets of real numbers on a number line.
4.2 Graph ordered pairs of numbers on the coordinate plane and interpret information related to these sets of points.
4.3 Find the distance between two points on a number line.
4.4 Graph a relation on the coordinate plane.
4.5 Distinguish between a relation and a function.
4.6 Graph a relation given an equation and domain.
4.7 Sketch a reasonable graph for a given relationship.
4.8 Interpret a graph in a real-world setting.
4.9 Use a computer or graphing calculator to explore the graphs of functions.
4.10 Compare ordered pairs to the line $y = x$ and interpret the results.

COMPETENCY GOAL 5: The learner will graph and use linear equations and inequalities.

5.1 Determine if data are behaving in a linear fashion.
5.2 Find the solution set of open sentences in two variables when given replacement sets for the variables.
5.3 Graph a linear equation in two variables.
5.4 Graph a line given its slope and y- intercept.
5.5 Find the slope of a non-vertical line given the graph of a line or an equation of the line or two points on the line.
5.6 Describe the slope in a real-world linear relationship using real-world terms.
5.7 Write the slope-intercept form of an equation of a line.
5.8 Write the equation of a line given the slope and one point on the line, or two points on the line.
5.9 Write the equation of a line which models a set of real data.
5.10 Use the line which models real data to make predictions.
5.11 Graph a linear inequality in two variables.

COMPETENCY GOAL 6: The learner will graph and solve systems of linear equations and inequalities.

6.1 Use a graph to find the solution of a pair of linear equations in two variables.
6.2 Graph the solution set of a system of linear inequalities in two variables.
6.3 Use a computer or graphics calculator to solve systems of linear equations.
6.4 Use the substitution method to find the solution of a pair of linear equations in two variables.
6.5 Use the addition or subtraction method to find the solution of a pair of linear equations in two variables.
6.6 Use multiplication with the addition or subtraction method to solve systems of linear equations.
6.7 Use systems of linear equations to solve problems.

COMPETENCY GOAL 7: The learner will perform operations with polynomials.

7.1 Add and subtract polynomials.
7.2 Multiply monomials.
7.3 Find an indicated power of a monomial.
7.4 Multiply a polynomial by a monomial.
7.5 Find the product of two binomials.
7.6 Multiply two polynomials.
7.7 Divide two monomials.
7.8 Divide a polynomial by a monomial.
7.9 Find a common monomial factor in a polynomial.
7.10 Factor the difference of two squares.
7.11 Factor a simple quadratic trinomial.

COMPETENCY GOAL 8: The learner will work with ratios, proportions, and percents.

8.1 Simplify ratios involving algebraic expressions.
8.2 Solve proportions.
8.3 Use ratios and proportions to solve problems.
8.4 Solve problems involving percents.

COMPETENCY GOAL 9: The learner will explore, graph and interpret nonlinear equations.

9.1 Graph a quadratic equation.
9.2 Use an automatic grapher to find the solution to a quadratic equation.
9.3 Solve a quadratic equation when one member is in factored form and the other member is zero.
9.4 Solve a second degree equation by factoring.
9.5 Use an automatic grapher to relate the solutions of quadratic equations and the *x*-intercepts.
9.6 Understand that the vertex provides the maximum or minimum value of the function.
9.7 Solve a quadratic equation in which a perfect square equals a constant.
9.8 Solve a quadratic equation by using the quadratic formula.
9.9 Use quadratic equations to solve problems.
9.10 Determine if a set of data represents an exponential function.
9.11 Use formulas, calculators and automatic graphers to explore and solve problems involving exponentials.

Technical Mathematics

COMPETENCY GOAL 1: The learner will use and read measuring devices and solve problems involving customary and metric units of measurement.

1.1 Find the mass/weight of objects using customary and metric units.
1.2 Find the volume/capacity of quantities using customary and metric units.
1.3 Compare and convert measurement units as needed to perform calculations, using a calculator when needed.
1.4 Use common measuring tools and solve problems involving the measurements.
1.5 Compare measurements to specified tolerances.
1.6 Use significant digits to indicate accuracy of measurement.
1.7 Select the appropriate measuring tool and units of measure to solve a problem.

COMPETENCY GOAL 2: The learner will use ratios and proportions to solve problems.

2.1 Read, interpret and compare ratios.
2.2 Recognize and write proportions from given information.
2.3 Distinguish between direct and indirect relationships.
2.4 Solve problems involving proportions.
2.5 Solve problems involving percents.
2.6 Interpret and construct maps and scale drawings.
2.7 Find the magnitude and direction of a vector.
2.8 Solve problems using signed numbers and vectors.

COMPETENCY GOAL 3: The learner will solve geometric problems using two- and three-dimensional shapes.

3.1 Identify polygons and their properties.
3.2 Calculate the perimeter, area, and circumference of polygons and circles.
3.3 Identify common three-dimensional shapes.
3.4 Calculate the surface area and the volume of three-dimensional shapes.
3.5 Solve problems involving common geometric properties.
3.6 Draw auxiliary diagrams to help solve for an unknown dimension or an unknown angle in both plane and solid shapes.
3.7 Solve both plane and solid geometry problems that involve a series of successive calculations.

COMPETENCY GOAL 4: The learner will analyze patterns and functions.

4.1 Decode and extend patterns.
4.2 Identify the domain and range of mathematical relations and functions.
4.3 Distinguish between mathematical relations and functions.
4.4 Using patterns and functions to solve problems at home and at work.

COMPETENCY GOAL 5: The learner will solve problems using triangle relationships.

5.1 Use the Pythagorean Theorem to find a side of a right triangle.
5.2 Use the characteristics of 3:4:5, 45° - 45° - 90° and 30° - 60° - 90° right triangles to solve practical problems.
5.3 Use the ratios for the sine, cosine, and tangent of an angle to solve problems that involve right triangles.
5.4 Solve problems involving the Law of Cosines and the Law of Sines.

COMPETENCY GOAL 6: The learner will graph and analyze trigonometric functions.

6.1 Draw a graph of sine and cosine waves.
6.2 Find the amplitude, wave length, period, and frequency of sine waves.
6.3 Find the phase shift between two sine waves.

COMPETENCY GOAL 7: The learner will use formulas to solve problems.

7.1 Translate a problem into an equation.
7.2 Evaluate algebraic expressions and formulas when replacement values are given.
7.3 Select an appropriate formula to solve a problem.
7.4 Solve problems with formulas using a calculator when needed.
7.5 Solve problems that involve powers and roots.

COMPETENCY GOAL 8: The learner will solve problems that involve nonlinear equations.

8.1 Recognize nonlinear equations (involving squares, square roots and reciprocals of the variable) and become familiar with their graphs.
8.2 Solve and graph a nonlinear equation.
8.3 Read values from the graph of a nonlinear equation.

COMPETENCY GOAL 9: The learner will solve problems involving systems of equations and inequalities.

9.1 Write equations and inequalities in terms of two unknowns.
9.2 Solve systems of equations.
9.3 Solve systems of inequalities.
9.4 Solve linear programming problems.

COMPETENCY GOAL 10: The learner will use statistics to analyze and solve real-world problems.

10.1 Distinguish between mean, mode, and median as measures of central tendency.
10.2 Calculate the mean, mode, and median for a set of data.
10.3 Draw and interpret a histogram to represent frequency distribution of data.
10.4 Distinguish between range, trend and standard deviation as measures of variability.
10.5 Interpret the characteristics of a normal curve.
10.6 Calculate the range and standard deviation to describe a set of data.

COMPETENCY GOAL 11: The learner will use probability to solve real-world problems.

11.1 Find the probability of some simple events.
11.2 Count the number of ways an event can happen.
11.3 Draw diagrams and charts to help find the probability of an event.

COMPETENCY GOAL 12: The learner will solve problems using computer technology.

12.1 Load and use simple spreadsheet templates to solve practical problems.
12.2 Use an appropriate graphics program to produce bar graphs, circle graphs, and line graphs.
12.3 Use an appropriate graphics program to graph linear and nonlinear functions.
12.4 Use an appropriate graphics program to graph one or more curves and determine such characteristics as their slope, roots and intersection points.
12.5 Use an appropriate graphics program to determine the effect of varying parameters in an equation on the shape of the curve representing the graphed equation.

Geometry

COMPETENCY GOAL 1: The learner will use concepts of points, lines, and planes in one, two and three dimensions.

1.1 Identify and name sets of points, such as line, ray, segment, and plane.
1.2 Draw representations of points, lines, and planes.
1.3 Identify interior and exterior points in two- and three-dimensional figures.
1.4 Find the coordinates of a point in a plane or in space.
1.5 Find the length of a segment on a line, in a plane, or in space.
1.6 Apply the segment addition postulate (definition of betweenness).
1.7 Use lengths to solve problems involving geometric probability.
1.8 Identify the midpoint of a given segment on a line, in a plane or in space.

COMPETENCY GOAL 2: The learner will write a valid proof using a variety of reasoning strategies.

2.1 Identify the structure of geometric deductive reasoning (undefined term, postulates, theorems).
2.2 State and use properties of equality and inequality.
2.3 Write and interpret statements in if-then form.
2.4 State the converse, inverse, and contrapositive of the conditional.
2.5 Write a proof using a flow diagram.
2.6 Write proofs using a two-column format.
2.7 Write proofs using a paragraph format.
2.8 Write indirect proofs.

COMPETENCY GOAL 3: The learner will use properties of angles, lines, and planes to solve problems and write proofs.

3.1 Use the definitions of adjacent, vertical, linear pair, complementary and supplementary angles to solve problems and write proofs.
3.2 Use the angle addition postualte to solve problems and write proofs.
3.3 Use the definition of angle bisectors to solve problems and write proofs.
3.4 Use the definitions of parallel lines, perpendicular lines, and perpendicular bisectors to solve problems and write proofs.
3.5 Use the relationships which exist between special pairs of angles formed by parallel lines and a transversal to solve problems and write proofs.
3.6 Use slopes to determine if two lines are parallel or perpendicular.
3.7 Write the equation of a line parallel or perpendicular to a given line through a given point.

COMPETENCY GOAL 4: The learner will use properties of polygons and polyhedra to solve problems and write proofs.

4.1 Model and describe convex polygons and regular polygons.
4.2 Use the measures of the interior and exterior angles of a convex polygon to solve problems.
4.3 Use the properties of proportions to solve problems.
4.4 Use properties of congruent and similar polygons to solve problems and write proofs.
4.5 Investigate a variety of transformations on polygons (tessellation, slide, rotation, flip) in a coordinate plane.
4.6 Model and describe regular and non-regular polyhedra.
4.7 Identify congruent or similar polyhedra
4.8 Determine coordinates of the vertices of polygons in a plane.
4.9 Use coordinate geometry to verify conclusions regarding polygons.

COMPETENCY GOAL 5: The learner will develop and use properties of quadrilaterals to solve problems and write proofs.

5.1 Use the definition and properties of parallelograms to solve problems and write proofs.
5.2 Develop and use properties of rectangles, rhombi, and squares to solve problems and write proofs.
5.3 Use the definition and properties of trapezoids and isosceles trap ezoids to solve problems and write proofs.
5.4 Given quadrilaterals in a coordinate plane, solve problems and construct proofs.

COMPETENCY GOAL 6: The learner will develop and use properties of triangles to solve problems and write proofs.

6.1 Classify triangles according to sides and angles.
6.2 Explore the relationships of the lengths of the sides of a triangle.
6.3 Investigate and apply theorems involving the interior and exterior angles of a triangle.
6.4 Use postulates and theorems to prove that two triangles and their corresponding parts are congruent.
6.5 Construct congruent triangles using congruency postulates and theorems.
6.6 Investigate and apply special theorems involving isosceles triangles.
6.7 Investigate and apply definitions and theorems involving altitudes, perpendicular bisectors, and medians.
6.8 Investigate and apply the theorem involving the segment joining the midpoints of two sides of a triangle.
6.9 Apply theorems involving segments divided proportionally.
6.10 Given triangles in the coordinate plane, solve problems and write proofs.

COMPETENCY GOAL 7: The learner will develop and use properties of right triangles to solve problems.

7.1 Find the geometric mean between a pair of numbers.
7.2 Use the Pythagorean Theorem and its converse to solve problems.
7.3 Use the relationships in a special right triangle to solve problems and write proofs.
7.4 State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle.
7.5 Use a calculator to apply the definitions of sine, cosine, and tangent to solve problems.
7.6 Use coordinate methods to solve problems and write proofs related to right triangles.

COMPETENCY GOAL 8: The learner will develop and use properties of circles and spheres to solve problems and write proofs.

8.1 Identify and use the definition of a circle and sets of points related to the circle.
8.2 Find the center of a circle which passes through three non-collinear points.
8.3 Describe the relationship between tangents and circles.
8.4 Apply the properties involving arcs and angles of circles.
8.5 Apply theorems that relate to the chords of a circle, excluding the product theorem.
8.6 Apply theorems that relate to the tangents, secants, and radii of a circle, excluding the product theorems.
8.7 Describe the relationship between the equation of a circle, its center and radius length.
8.8 Discuss the relationships of congruent, similar, and concentric circles.
8.9 Explore spheres and sets of points related to the sphere.

COMPETENCY GOAL 9: The learner will understand and use perimeter, area, and volume formulas.

9.1 Find the perimeter of a geometric figure.
9.2 Find the area of a triangle, parallelogram, trapezoid, and rectangle.
9.3 Find the ratio of the perimeters, areas, and volumes of similar geometric figures.
9.4 Find the circumference and area of a circle.
9.5 Compute arc lengths and the area of sectors of a circle.
9.6 Use areas to solve problems involving geometric probability.
9.7 Find the lateral area, total area, and volume of a right prism, pyramid, right circular cylinder and cone.
9.8 Compute the surface area and volume of a sphere to solve problems.
9.9 Use an automatic grapher to solve "Max-Min" problems.

Algebra II

COMPETENCY GOAL 1: The learner will solve, graph and use equations and inequalities.

1.1 Solve literal equations and formulas for a specified variable.
1.2 Use an automatic grapher to estimate solutions of linear and absolute value equations and inequalities.
1.3 Solve and graph equations and inequalities involving absolute value.
1.4 Interpret the slope and intercepts of a line.
1.5 Apply the concepts of parallel and perpendicular lines as determined by a comparison of their slopes.
1.6 Write and use an equation of a line which models a set of linear data.
1.7 Use an automatic grapher to fit a line to a set of linear data. Interpret the slope, intercepts and quality of fit.

COMPETENCY GOAL 2: The learner will demonstrate an understanding of relations and functions.

2.1 Determine if a given relation is a function.
2.2 Identify the domain and range of a relation.
2.3 Use function notation.
2.4 Graph relations and functions with and without an automatic grapher.
2.5 Find the zeros of a function by examining a graph.
2.6 Solve an inequality by examining the graph of the function.
2.7 Communicate graphically, algebraically and verbally real-world phenomena as functions.
2.8 Find the composition of functions.
2.9 Use iterative definitions of functions.

COMPETENCY GOAL 3: The learner will operate with matrices.

3.1 Organize data into an array or matrix.
3.2 Add and subtract matrices with and without calculators.
3.3 Use scalar multiplication and multiply matrices with and without calculators.
3.4 Solve real-world problems using matrices.
3.5 Write and solve systems of linear equations in matrix form.
3.6 Solve matrix equations of the form $AX=B$ using calcultors or computers.

COMPETENCY GOAL 4: The learner will graph and solve systems of equations and inequalities.

4.1 Solve systems of two equations graphically with and without an automatic grapher.
4.2 Solve systems of two equations in two variables.
4.3 Solve systems of three equations in three variables.
4.4 Use systems of equations and inequalities to solve problems.
4.5 Solve systems of inequalities by graphing.
4.6 Use linear programming to solve real-world problems.

COMPETENCY GOAL 5: The learner will perform operations and solve problems with polynomials.

5.1 Divide one polynomial by another of a lower degree.
5.2 Use synthetic division to divide a polynomial by a linear binomial.
5.3 Factor polynomials completely.
5.4 Use factoring to solve polynomial equations.
5.5 Use polynomial equations to solve problems.
5.6 Expand powers of binomials using Pascal's triangle or the binomial theorem.
5.7 Write a polynomial equation given its solutions.

COMPETENCY GOAL 6: The learner will use rational expressions to solve problems.

6.1 Simplify ratios involving algebraic expressions.
6.2 Use expressions involving negative exponents.
6.3 Find sums and differences of rational algebraic expressions.
6.4 Find products and quotients of rational algebraic expressions.
6.5 Simplify complex fractions.
6.6 Solve fractional equations.
6.7 Solve problems involving fractional equations.
6.8 Solve problems of direct and inverse variation.
6.9 Use joint and combined variation to solve problems.

COMPETENCY GOAL 7: The learner will solve problems with quadratic equations and inequalities.

7.1 Complete the square to solve quadratic equations.
7.2 Use the quadratic formula to solve quadratic equations.
7.3 Define complex numbers and perform basic operations with them.
7.4 Solve quadratic inequalities.
7.5 Determine the solutions of quadratic and other polynomial equations using an automatic grapher.
7.6 Solve problems using quadratic equations and inequalities.
7.7 Interpret the maximum and minimum values of a quadratic function.
7.8 Use the discriminant of a quadratic equation to determine the nature of the roots and the number of x-intercepts of the graph.
7.9 Explore the relationship between coefficients and solutions of a quadratic equation.
7.10 Solve equations which contain radical expressions.
7.11 Explore complex numbers as solutions to quadratic equations.

COMPETENCY GOAL 8: The learner will use analytic geometry to solve problems.

8.1 Use the distance and midpoint formulas.
8.2 Write the equations of and graph circles and parabolas given their geometric properties.
8.3 Explore the equations of and graph ellipses and hyperbolas.

COMPETENCY GOAL 9: The learner will solve problems involving logarithmic and exponential functions.

9.1 Use expressions involving fractional exponents.
9.2 Write an exponential function of the form $f(x) = a \bullet b^x$ given the base and a point.
9.3 Graph exponential functions of the form $f(x) = a \bullet b^x$.
9.4 Use exponential equations of the form $f(x) = (1+ r)^x$ where r is given as a rate of growth.
9.5 Apply the definition of logarithms.
9.6 Use properties of logarithms and exponents.
9.7 Use logarithms to solve expressions of the form $a \bullet b^x = c$ for x.

COMPETENCY GOAL 10: The learner will solve problems involving sequences and series.

10.1 Generate the terms of an arithmetic series by iteration.
10.2 Use a calculator or computer to generate the terms of a geometric series by iteration.
10.3 Use summation notation to describe the sums in a series.

Advanced Mathematics

COMPETENCY GOAL 1: The learner will model real-world phenomena using techniques of data analysis.

1.1 Recognize a mathematical model of linear, quadratic, exponential, trigonometric, and logarithmic functions.
1.2 Use a scatter plot to determine if a given model is appropriate.
1.3 Find the equation of a line that models a bivariate linear data set.
1.4 Use the linear least squares method to fit linear data.
1.5 Interpret the slope and y-intercept of a linear model.
1.6 Determine the goodness-of-fit of a model using residuals and/or correlation coefficients.
1.7 Use models where appropriate to draw conclusions or make predictions.

COMPETENCY GOAL 2: The learner will graph, transform, use as mathematical models, and compose basic functions.

2.1 Sketch graphs of the basic functions (constant, linear, quadratic, cubic, square root, absolute value, reciprocal, trigonometric, exponential, logarithmic, and greatest integer).
2.2 Compare information given by local behavior versus global behavior on graphs produced on an automatic grapher.
2.3 Find the domain of a function.
2.4 Estimate the range of a function.
2.5 Determine the symmetry of a given graph.
2.6 Identify continuous and discontinuous functions and locate points of discontinuity.
2.7 Graph transformations and combinations of transformations for all basic functions.
2.8 Find coordinates of zeros and maximum or minimum points of a given function.
2.9 Write the equation of a function from a description of its behavior.
2.10 Solve inequalities using an automatic grapher or number line analysis.
2.11 Compose two functions and find the domain of the composition.
2.12 Analyze a function by decomposing it into simpler functions.
2.13 Find the inverse of a function and the domain of the inverse.

COMPETENCY GOAL 3: The learner will graph and use as models polynomial and rational functions.

3.1 Find zeros and factors of polynomials algebraically (where appropriate) and/or using an automatic grapher.
3.2 Estimate the turning points of polynomial functions using an automatic grapher.
3.3 Find the zeros and vertical asymptotes of a rational function through analysis of the polynomials in the numerator and denominator.
3.4 Find the horizontal asymptote of a rational function by comparing the growth of the polynomials in the numerator and denominator.
3.5 Sketch the graph of a rational function labeling the horizontal and vertical asymptotes and the x- and y-intercepts.

COMPETENCY GOAL 4: **The learner will graph, transform, and solve problems involving the exponential and logarithmic functions.**

4.1 Model growth using transformations of: $f(x) = ab^x$ and $f(x) = (1 + r)^x$.
4.2 Define and use the logarithmic function as the inverse of the exponential function.
4.3 Find the domain of exponential and logarithmic functions.
4.4 Sketch and investigate the graphs of composition of exponential and logarithmic functions with each other and with other basic functions.
4.5 Solve equations containing exponential and logarithmic functions, using real-world problems.

COMPETENCY GOAL 5: **The learner will use techniques of data analysis to model nonlinear data from real-world phenomena.**

5.1 Linearize data using concepts of composition and inverses in order to find a model for data.
5.2 Rewrite the linear equation that models linearized data to fit the original curved data.
5.3 Discuss the goodness-of-fit of the model used to represent data.
5.4 Use log-log re-expression to linearize date of the form $y = ax^b$ by plotting $(\log x, \log y)$.

COMPETENCY GOAL 6: **The learner will graph and transform trigonometric functions, solve trigonometric equations and inequalities, and use trigonometric functions as mathematical models.**

6.1 Use trigonometric functions to model periodic phenomena.
6.2 Express the tangent, cotangent, secant, and cosecant functions in terms of sine and cosine.
6.3 Sketch a graph of each of the six trigonometric functions and identify the period of each.
6.4 Recognize and graph transformations of each of the six trigonometric functions.
6.5 Use graphs to develop, recognize, and validate trigonometric identities.
6.6 Understand how the values of sine and cosine are represented on the unit circle.
6.7 Use the symmetry of the unit circle to develop, recognize, and vali date trigonometric identities.
6.8 Understand the relationship between trigonometry in degree mode and trigonometry in radian mode.
6.9 Find the radian measure that corresponds to a given angle or arc length.
6.10 Solve trigonometric equations and inequalities either algebraically or using an automatic grapher.
6.11 Find values of inverse trigonometric functions, applying appropriate domain and range restrictions.
6.12 Evaluate and graph compositions of trigonometric and inverse trigonometric functions.
6.13 Use the Law of Sines and Law of Cosines to solve problems involving triangles and vectors.

6.14 Convert complex numbers from rectangular form to polar form.
6.15 Convert complex numbers from polar form to rectangular form.
6.16 Use DeMoivre's Theorem to find roots and powers of complex numbers.
6.17 Use an automatic grapher to explore polar equations.

COMPETENCY GOAL 7: The learner will use discrete mathematics concepts to solve problems.

7.1 Follow an algorithm.
7.2 Use operations with matrices and the inverse of a matrix to solve real-world problems.
7.3 Use permutations and combinations to solve problems.
7.4 Classify events as independent or dependent or mutually exclusive.
7.5 Interpret data in terms of mean and standard deviation.
7.6 Recognize and use the normal distribution curve to interpret data.
7.7 Find indicated terms in sequences.
7.8 Find the sum of a finite series.
7.9 Find the limit of an infinite sequence.
7.10 Find whether a given series converges or diverges.
7.11 Use mathematical induction to establish a generalization.

Advanced Placement Calculus

COMPETENCY GOAL 1: Use elementary functions, including algebraic, trigonometric, exponential, and logarithmic.

1.1 Define a function and relate the idea of function to real situations.
1.2 Find the domain and range of a function with and without an automatic grapher.
1.3 Find the sum, product, and quotient of two functions.
1.4 Find the composition of two functions and decompose two functions.
1.5 Find the domain of a composition of two functions.
1.6 Find and apply the absolute value of a function.
1.7 Find and use the inverse of a function.
1.8 Determine if a function is odd, even, or neither odd nor even.
1.9 Determine periodicity and amplitude of a function.
1.10 Describe the symmetry of a function.
1.11 Find the asymptotes of a function.
1.12 Find the zeros of a function.
1.13 Use a^x and $\log_a x$ and their inverse relationships.

COMPETENCY GOAL 2: Find and use limits of various functions.

2.1 Find limits of functions by direct substitution.
2.2 Find the limit of a quotient of indeterminant form.
2.3 Find the limit of a function as the independent variable approaches infinity.
2.4 Find the limit of a function by examining its graph.
2.5 Recognize the limit of special functions.
2.6 Recognize functions that have non-existent limits.

COMPETENCY GOAL 3: Use the definition of continuity.

3.1 Apply the definition of continuity and find excluded values of a discontinuous function.
3.2 Use the statement: "If f is continuous on $[a, b]$, then f has a maximum and minimum on $[a, b]$."
3.3 Apply the intermediate value theorem.

COMPETENCY GOAL 4: Use the concepts of differential calculus.

4.1 Understand the concept of the derivative.
4.2 State and apply the definitions of the derivative.
4.3 Find the derivatives of elementary functions.
4.4 Find the derivative of a sum, product, and quotient.
4.5 Find the derivative of a composite function (chain rule).
4.6 Find the derivative of an implicity defined function.
4.7 Find the derivative of the inverse of a function.
4.8 Use logarithmic differentiation.
4.9 Find derivatives of higher order.
4.10 Apply the Mean Value Theorem.
4.11 Use the relation between differentiability and continuity.
4.12 Use L'Hopital's rule.

COMPETENCY GOAL 5: Apply the concepts of a derivative.

5.1 Find the slope of a curve.
5.2 Find the tangent line to a curve.
5.3 Find the normal line to a curve.
5.4 Use the differential to approximate values.
5.5 Use Newton's Method to approximate the zeros of a function.
5.6 Determine where a function is increasing and where it is decreasing.
5.7 Find critical points, relative (local) and absolute maximum and minimum points.
5.8 Determine the concavity and points of inflection of a function.
5.9 Use the graph of the derivative of a function to identify information about the function.
5.10 Solve extreme value problems.
5.11 Find the velocity and the acceleration of a particle moving along a line.
5.12 Find average rates of change.
5.13 Find instantaneous rates of change.
5.14 Determine related rates of change.

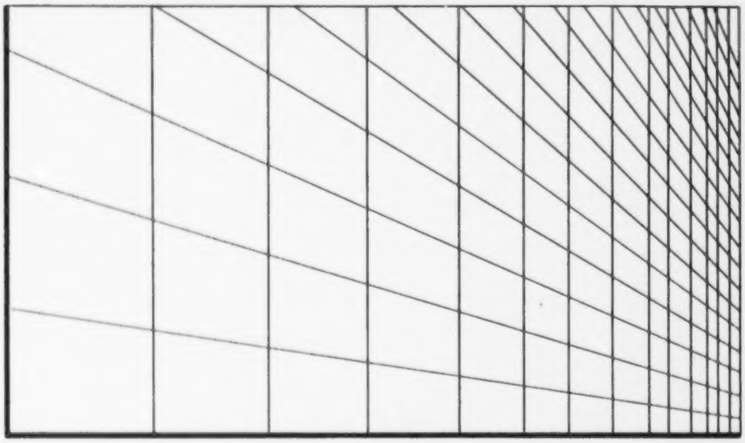
COMPETENCY GOAL 6: Use the concepts of integral calculus.

6.1 Find antiderivatives.
6.2 Find the distance and velocity from acceleration with initial conditions.
6.3 Solve simple first order differentiable equations.
6.4 Apply solutions of $y' = ky$ to growth and decay problems.
6.5 Use basic integration formulas.
6.6 Use substitution to integrate.
6.7 Do simple integration by parts.
6.8 Approximate the area under a curve using rectangles or trapezoids.
6.9 Recognize the definitions of the definite integral as the limit of a sum.
6.10 Understand the concept of the definite integral.
6.11 Apply properties of the definite integral.
6.12 Use the Fundamental Theorem.

COMPETENCY GOAL 7: Apply the concept of the definite integral.

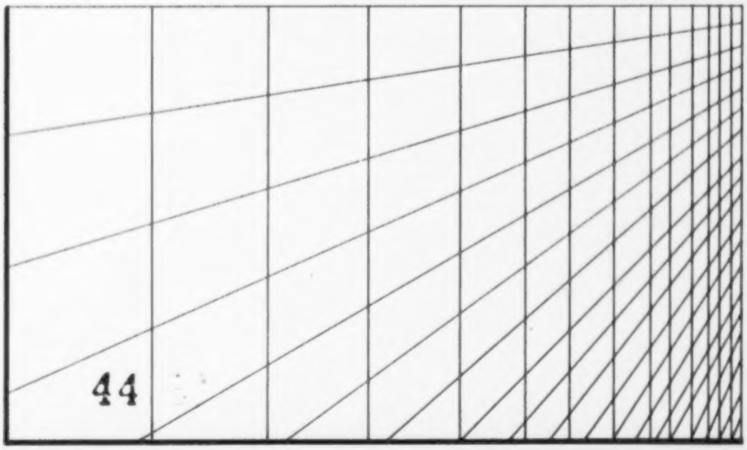
7.1 Find the average (mean) value of a function of an interval.
7.2 Find the area between curves.
7.3 Find the volume of a solid of revolution about the axes or lines parallel to the axes.

NOTES



Teacher Handbook Component

Mathematics



MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will demonstrate an understanding of and use real numbers to solve problems.

OBJECTIVES	SAMPLE MEASURES
1.1 Read, write, and use numbers in various forms, including fractions, decimals, percents, and exponential notation.	1.1.1 Which is not equivalent to $\frac{1}{2}$? (a) 50% (b) $\frac{5}{100}$ (c) 0.5 (d) $\frac{50}{100}$ 1.1.2 Write $(3 \times 10^4) + (2 \times 10^3) + (5 \times 10^0) + (6 \times 10^{-1})$ in standard notation.
1.2 Write numbers in scientific notation, using a calculator when appropriate.	1.2.1 Write the number 0.0003 using scientific notation. (a) 3.0×10^{-4} (b) 0.3×10^4 (c) 3.0×10^4 (d) 3.0×10^5 1.2.2 Write the number 30,080 in scientific notation. (a) use pencil and paper (b) use a calculator 1.2.3 Give a reason for the need to write numbers in scientific notation.
1.3 Use patterns to explore the tests for divisibility.	1.3.1 Which number below is divisible by both 2 and 3? (a) 12 (b) 99 (c) 130 (d) 56 1.3.2 Replace the $\underline{?}$ with a digit so that the number is divisible by 4. $74\underline{?}$

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 1: The learner will demonstrate an understanding of and use real numbers to solve problems.

OBJECTIVES	SAMPLE MEASURES
1.4 Use exponential notation to express the prime factorization of a number.	1.4.1 Which is the prime factorization of 150? (a) $2 \cdot 3^2 \cdot 5$ (b) $2 \cdot 3 \cdot 5^2$ (c) $2^2 \cdot 3 \cdot 5$ (d) $2 \cdot 3 \cdot 25$
	1.4.2 Write the prime factorization of 1450.
1.5 Find the greatest common factor (GCF) and the least common multiple (LCM) of two or more numbers.	1.5.1 What is the greatest common factor of 24 and 36? (a) 12 (b) 72 (c) 6 (d) 60
	1.5.2 What is the least common multiple of 18 and 24?
1.6 Compare two or more rational numbers.	1.6.1 Replace the $\underline{?}$ with <, >, or = to make the following a true statement. $\frac{2}{3} \underline{?} \frac{3}{4}$ (a) < (b) > (c) = (d) none of these
	1.6.2 Which number is the largest? $-\frac{3}{4}, -\frac{5}{6}, -\frac{7}{8}$
	1.6.3 Use $\frac{3}{4}, \frac{5}{6}$, and > to write a true statement.
	1.6.4 Describe a way to remember which symbol represents "is less than" and which represents "is more than".

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will demonstrate an understanding of and use real numbers to solve problems.

OBJECTIVES	SAMPLE MEASURES
1.7 Estimate sums, differences, products, and quotients of real numbers.	1.7.1 Estimate the product of the following two factors by rounding each factor to the nearest whole number: 127.36 x 80.9 (a) 10,287 (b) 10,160 (c) 10,319 (d) 10,368 1.7.2 Use compatible numbers to estimate the quotient in the following problem: 358.8 ÷ 11.59 1.7.3 Choose the best estimate for the answer: $\begin{array}{r} 2\frac{1}{8} \\ -\ 1\frac{3}{4} \\ \hline \end{array}$ (a) almost one (b) more than one (c) less than $\frac{1}{2}$ (d) none of these 1.7.4 Estimate: $6\frac{3}{4} \times 3\frac{1}{2}$
1.8 Use a table of squares or a calculator to find the square root of a number.	1.8.1 Approximate $\sqrt{4100}$ to the nearest hundredth using a calculator. (a) 64.03 (b) 205 (c) 60.43 (d) 20.50 1.8.2 Find $\sqrt{24025}$.

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 1: The learner will demonstrate an understanding of and use real numbers to solve problems.

OBJECTIVES	SAMPLE MEASURES
1.9 Estimate the square root of a number.	1.9.1 Which is the best whole number estimate for $\sqrt{32}$? (a) 5 (b) 6 (c) 30 (d) 32 1.9.2 Use 15, $\sqrt{144}$, and $<$ to write a true statement.
1.10 Demonstrate an understanding of the relationships between fractions, decimals, percents, and ratios.	1.10.1 Which is the largest number in this list? (a) 75% (b) 0.8 (c) $\frac{7}{8}$ (d) 3 to 5 1.10.2 Order the numbers in this list from least to the greatest: $\frac{1}{2}$, 40%, 0.6, 9 to 10.
1.11 Relate common fractions to frequently used percents; estimate and calculate using these percents.	1.11.1 Represent $\frac{1}{2}$ as a percent. (a) 1.2% (b) 2% (c) 50% (d) 5% 1.11.2 Estimate 24% of 16.
1.12 Find the missing term of a proportion.	1.12.1 Solve: $\frac{12x}{25} = \frac{36}{15}$ (a) 3 (b) 5 (c) 2 (d) $\frac{1}{5}$ 1.12.2 Solve: $\frac{x}{50} = \frac{18}{30}$

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will demonstrate an understanding of and use real numbers to solve problems.

OBJECTIVES	SAMPLE MEASURES
1.13 Find the percent of a given number.	1.13.1 What is 125% of 8? (a) 1 (b) 10 (c) 100 (d) 1000
	1.13.2 What is 4.5% of 600?
	1.13.3 Explain how you can use your calculator to find 11.25% of 1600.
1.14 Find what percent one number is of another.	1.14.1 What percent of 25 is 16? (a) 16% (b) 25% (c) 64% (d) 156.25%
	1.14.2 8 is what percent of 40?
1.15 Find a number when a percent of it is given.	1.15.1 17 is 20% of what number? (a) 85 (b) 34 (c) 3.4 (d) 0.85
	1.15.2 24 is 3% of what number?
1.16 Write fractions as equivalent, terminating, or repeating decimals.	1.16.1 Write $\frac{5}{12}$ as a decimal. (a) 0.416 (b) $0.41\overline{6}$ (c) $0.4\overline{16}$ (d) $0.\overline{416}$
	1.16.2 Explain how to determine if $\frac{3}{4}$ is equivalent to $\frac{39}{52}$.

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 1: The learner will demonstrate an understanding of and use real numbers to solve problems.

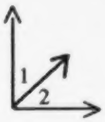
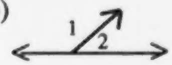
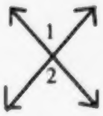
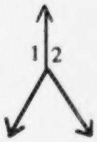
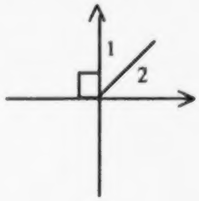
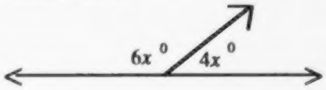
OBJECTIVES	SAMPLE MEASURES
1.17 Write terminating or repeating decimals as equivalent fractions.	1.17.1 Write 0.75 as a fraction in lowest terms. (a) $7\frac{1}{2}$ (b) $1\frac{1}{3}$ (c) $\frac{7}{5}$ (d) $\frac{3}{4}$
	1.17.2 Write $0.\overline{6}$ as a fraction in lowest terms.
1.18 Find the absolute value of a real number.	1.18.1 Simplify: $\lvert -6 \rvert + \lvert 8 \rvert$ (a) 2 (b) 14 (c) –2 (d) –14
	1.18.2 Explain why the absolute value of a number is always a positive value.
1.19 Use laws of exponents to write expressions in equivalent forms.	1.19.1 Simplify: $(2x^2)(3x^4)$ (a) $6x^8$ (b) $5x^6$ (c) $5x^8$ (d) $6x^6$
	1.19.2 Simplify: $2^3 \cdot x^3 \cdot 3^2 \cdot x^2$

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of and use properties and relationships of geometry.

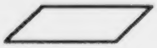
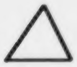
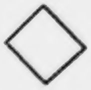
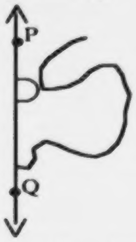
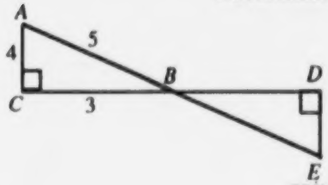
OBJECTIVES	SAMPLE MEASURES
2.1 Classify angles (interior, exterior, vertical, complementary, supplementary) and pairs of lines (parallel, perpendicular, skew) and solve problems involving them.	2.1.1 In which figure are angles 1 and 2 supplementary? (a)  (b)  (c)  (d) 
	2.1.2 In the figure, $m\angle 1 = 30^\circ$. Find $m\angle 2$. 
	2.1.3 Solve for x: 
	2.1.4 Draw and label a diagram to show each of the following: (a) Angles *XYZ* and *ZYW* are adjacent, complementary angles. (b) Angles *RST* and *TSW* are adjacent, supplementary angles. (c) Draw $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, and transversal *t*. Label the interior angles as angles 1, 2, 3, and 4. (d) Draw ΔEFG with exterior angle *EGH*.

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of and use properties and relationships of geometry.

OBJECTIVES	SAMPLE MEASURES
2.2 Identify and distinguish among similar, congruent, and symmetric figures; name corresponding parts.	2.2.1 Which figure appears to be congruent to △ ? (a)  (b)  (c)  (d) not given
	2.2.2 Are all squares similar? Explain your answer.
	2.2.3 Complete the figure so that $\overleftrightarrow{PQ}$ is a line of symmetry. 
	2.2.4 Discuss the following statements: "All congruent figures are similar." "All similar figures are congruent."
	2.2.5 Given $\Delta ABC \cong \Delta EBD$, how long is $\overline{DE}$? (a) 3 (b) 4 (c) 5 (d) cannot be determined 

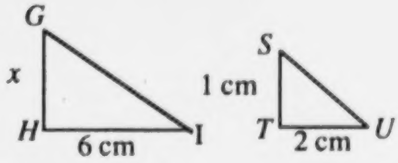
MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of and use properties and relationships of geometry.

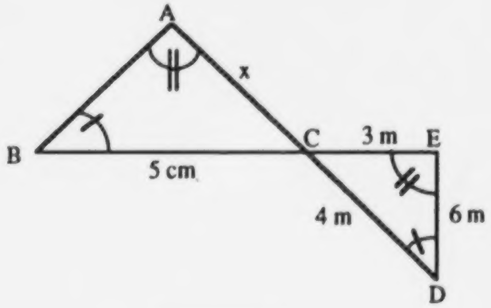
OBJECTIVES	SAMPLE MEASURES
2.3 Find the lengths of the sides of similar figures.	2.3.1 $\Delta GHI \sim \Delta STU$. Find x.

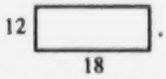

G
x
H
6 cm
I
S
1 cm
T
2 cm
U

(a) $x = 1$ cm (b) $x = 2$ cm

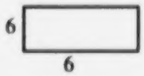
(c) $x = 3$ cm (d) $x = 4$ cm

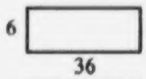
2.3.2 Study the drawing and solve for x.

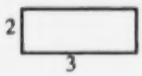

A
x
B
5 cm
C
3 m
E
4 m
6 m
D

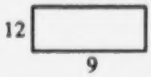
2.3.3 Given: 

. Which of the following is similar to the given rectangle?







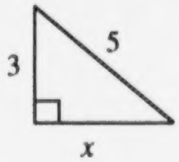


MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of and use properties and relationships of geometry.

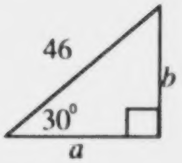
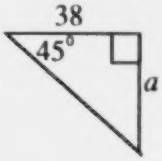
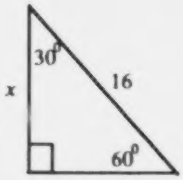
OBJECTIVES	SAMPLE MEASURES
2.4 Use the Pythagorean Theorem to find the missing side of a right triangle and use a table of square roots, calculator or mental math to approximate square roots.	2.4.1 In the triangle below, use the Pythagorean Theorem to find x:  (a) 2 (b) 4 (c) 5.8 (d) 8 2.4.2 A sign on a building is 12 feet above the ground. You have a 13-foot ladder. Use the Pythagorean Theorem to explain why the base of the ladder must be five feet or closer to the wall. 2.4.3 The lengths of the legs of a right triangle are 12 m and 16 m. Find the length of the hypotenuse. (a) 20 m (b) 12 m (c) 14 m (d) 16 m 2.4.4 If the hypotenuse of a right triangle is 15 inches and one leg is 8 inches, find the length of the other leg.

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of and use properties and relationships of geometry.

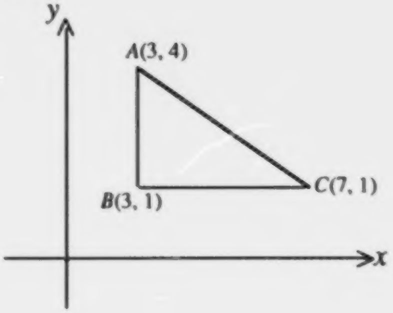
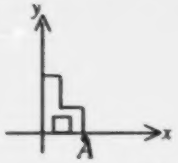
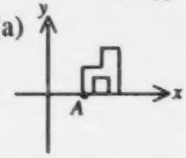
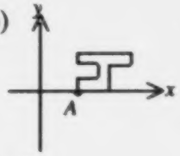
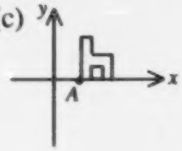
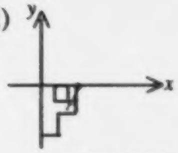
OBJECTIVES	SAMPLE MEASURES
2.5 Find the lengths of the missing sides of $30^{0}-60^{0}-90^{0}$ and $45^{0}-45^{0}-90^{0}$ right triangles given the length of one side.	2.5.1 Find *b*.  (a) 92 (b) 76 (c) 23 (d) 16
	2.5.2 Find *a*. 
	2.5.3 Approximate *x* to the nearest tenth. 

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of and use properties and relationships of geometry.

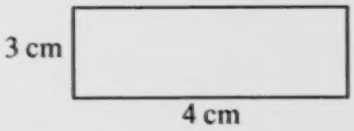
OBJECTIVES	SAMPLE MEASURES
2.6 Graph similar geometric figures on a coordinate plane.	2.6.1 A translation moves ΔABC 2 units to the right and 4 units down. The image is $\Delta A'B'C'$. Write the coordinates of each vertex of $\Delta A'B'C'$.  2.6.2 If the figure is rotated about point A, the resulting figure could be:  (a)  (b)  (c)  (d) 

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of and use properties and relationships of geometry.

OBJECTIVES	SAMPLE MEASURES
2.7 Solve problems that relate geometric concepts to real world situations.	2.7.1 Find the length of the room below in meters if the scale of the drawing is 1 cm:1.5 m.  (a) 4.5 m (b) 6 m (c) 7 m (d) none of these
	2.7.2 In a house plan the actual height of 6 feet is represented by 2 inches. If the actual length of the house is 120 feet, what is the length of the house in the drawing?
	2.7.3 If you had 20' of fencing, describe the shape you would use to give your pet the maximum amount of play room. Justify why it would be better than other shapes.

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 3: The learner will demonstrate an understanding of the language of algebra.

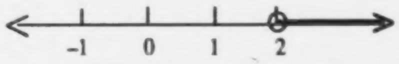
OBJECTIVES	SAMPLE MEASURES
3.1 Use order of operations to simplify numerical expressions, verifying the results with a calculator or computer.	3.1.1 Simplify: $23 - 3(5 + 2^2)$ (a) 180 (b) -4 (c) 4 (d) -124
	3.1.2 Simplify: $2^3 + (7 - 3) \div 2 + (7 - 2)^2$
	3.1.3 When simplifying, explain why $3^2 + 6 \div 3 \neq 5$.
	3.1.4 Write an expression involving multiplication and addition in which you should add first.
3.2 Identify and use the commutative, associative, and distributive properties.	3.2.1 $8 \cdot 5\frac{1}{2} = (8 \cdot 5) + (8 \cdot \frac{1}{2})$ is an example of which property? (a) Associative property of addition (b) Distributive property (c) Associative property of multiplication (d) Commutative property of addition
	3.2.2 Use number properties to find (0.5) (2.7) (2.0) mentally.
	3.2.3 Sally bought six plates for \$3.25 each and six bowls for \$1.25 each. Write two different expressions that Sally can use to find the total cost.
	3.2.4 Use the distributive property to add $3x$ and $7x$.

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will demonstrate an understanding of the language of algebra.

OBJECTIVES	SAMPLE MEASURES
3.3 Evaluate algebraic expressions.	3.3.1 Evaluate the expression $3x + 4y$ if $x = 3$ and $y = -5$. (a) -12 (b) 29 (c) -11 (d) -3
	3.3.2 Write an expression using x and y that equals 4 if $x = 5$ and $y = -2$.
	3.3.3 Evaluate the expression below if $a = 2$, $b = 3$, and $c = -2$: $ab - c$
	3.3.4 Evaluate $3ab^2 + 4c$ if $a = 2$, $b = 3$, and $c = -2$.
3.4 Solve simple equations algebraically.	3.4.1 Solve: $7 + 3t = -20$ (a) $-4\frac{1}{3}$ (b) -9 (c) -2 (d) 2
	3.4.2 Solve: $-7 - \frac{x}{5} = -10$
	3.4.3 Solve: $\frac{2}{3}n + 12 = 18$ (a) 27 (b) 9 (c) 6 (d) 18
	3.4.4 Solve: $25 - 6n = 7$
	3.4.5 State two ways to solve $4x + 12 = 28$.
3.5 Solve simple inequalities and graph their solutions.	3.5.1 Solve: $3x + 1 < 10$
	3.5.2 Write an inequality which has the solution set graphed below: 
	3.5.3 Explain how the solutions of $x + 4 = -2$ and $x + 4 > -2$ differ.

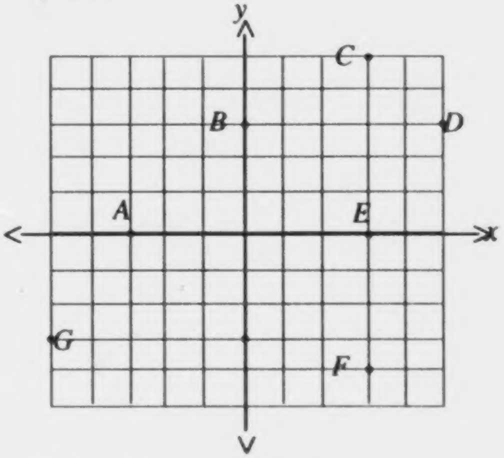
MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 3: The learner will demonstrate an understanding of the language of algebra.

OBJECTIVES	SAMPLE MEASURES
3.6 Use graphs and tables to represent relations and functions of ordered pairs.	3.6.1 State the letter which represents point *C*.



(a) (3, 5) (b) (–5, 3)

(c) (3, 0) (d) (0, 3)

3.6.2 Graph the relation shown in the table:

x	*y*
–1	–1
0	1
2	5
3	7

3.6.3 Describe the relationship between *x* any *y* in the following table:

x	–2	–1	0	1	2
y	–6	–3	0	3	6

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will demonstrate an understanding of the language of algebra.

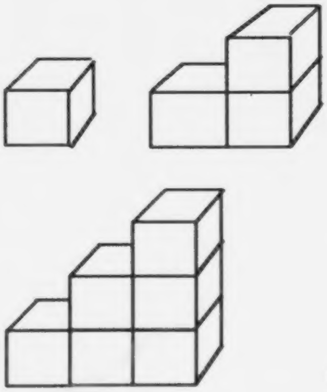
OBJECTIVES	SAMPLE MEASURES
3.7 Describe, extend, analyze and create a variety of geometric and numerical patterns to investigate relationships and solve problems.	3.7.1 Josh made a spinach pie and gave one half of it to his grandparents. The part that remained was cut into thirds and two of the pieces were eaten. The last piece was cut into halves and one piece was eaten. What part of the original pie was the last piece?
	3.7.2 Jane told three friends a secret code. Each of these three told four other friends, each of whom told five other friends. How many people knew about the code?
	3.7.3 What is the product of the first 10 terms of the sequence: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$? (a) $\frac{1}{10}$ (b) $\frac{1}{11}$ (c) $\frac{24}{120}$ (d) 11
	3.7.4 As the hands of a clock move from 6 o'clock in the morning to 6 o'clock in the evening, how many times do the hands form a right angle?

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 3: The learner will demonstrate an understanding of the language of algebra.

OBJECTIVES	SAMPLE MEASURES
3.7 Describe, extend, analyze and create a variety of geometric and numerical patterns to investigate relationships and solve problems.	3.7.5 In the following pattern, the volume of one small cube is one cubic unit. What would be the volume of the ninth figure? 
3.8 Given an equation, formulate a problem.	3.8.1 Write a realistic problem which could be solved with this equation. $3n + 4 = 13.$ 3.8.2 Create a problem for the following equation: $12x + 170 = 770$

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will demonstrate an understanding of the language of algebra.

OBJECTIVES	SAMPLE MEASURES
3.9 Write an equation which could be used to solve a given problem.	3.9.1 The sum of the number x and seven is fourteen. Write the sentence in equation form. (a) $x + 7 = 14$ (b) $x = 7 + 14$ (c) $7x = 14$ (d) $x + 14 = 7$ 3.9.2 If three is added to five times the number n, the sum is 18. Write an equation to represent the sentence. 3.9.3 A group of friends rode to a concert together. Each ticket cost $10. They had to pay $5 for parking. The total cost was $65. How many people went to the concert? Which equation could be used to solve this? (a) $10n + 5n = 65$ (b) $10n + 5 = 65$ (c) $5n + 10 = 65$ (d) not enough information 3.9.4 Bob saves $x per month for three months. His father gives him $40 and he has enough money to buy the $115 tennis shoes he wants. Which equation shows this information? (a) $3x + 25 = 115$ (b) $115 - 40 = 3x$ (c) $3x + 40 = 115$

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 3: The learner will demonstrate an understanding of the language of algebra.

OBJECTIVES	SAMPLE MEASURES
3.10 Given a rule or function that describes a linear equation, generate ordered pairs and graph the equation.	3.10.1 Which equation describes this set of ordered pairs? $(-1, -7), (0, -4), (1, -1), (2, 2)$ (a) $4x - y = 4$ (b) $3x + y = 2$ (c) $x + y = 4$ (d) $3x - y = 4$ 3.10.2 Find three ordered pairs that satisfy the equation $2x + y = 8$ and graph the equation on graph paper.

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will demonstrate an understanding of the language of algebra.

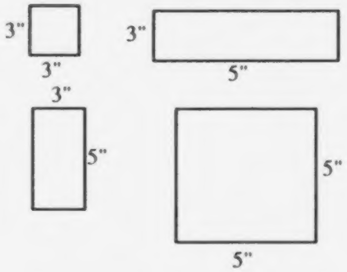
OBJECTIVES	SAMPLE MEASURES
3.11 Given a formula make appropriate substitutions and solve for one unknown.	3.11.1 Use the formula $D = r \cdot t$ to find the time (t) when $r = 50$ m/h and $D = 175$ miles. (a) 3 hr 50 min (b) 3 hr 30 min (c) 8750 hr (d) 8750 min 3.11.2 Mary used the formula $D = r \cdot t$ to determine how long it should take her to drive 250 miles at 50 miles/hour. She got 200 hours for an answer. Explain her error and how to get the correct answer. 3.11.3 Use the formula $V = lwh$ to find the height of a rectangular prism with volume = 96m^3, width = 6 m, and length = 2 m. (a) 84m (b) 12m (c) 8m (d) 1152m 3.11.4 The area formula for a triangle is: $A = \frac{1}{2}bh$ Find the area of a triangle with a base of 4 in. and height of 2.5 in. 3.11.5 The area formula for a trapezoid is: $A = \frac{1}{2}h(b_1 + b_2)$ If the height of a trapezoid is 5 cm and the bases measure 4 cm and 3 cm, find the area. (a) 35 cm^2 (b) 17.5 cm^2 (c) 30 cm^2 (d) 25 cm^2

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 4: The learner will demonstrate an understanding and use of measurement.

OBJECTIVES	SAMPLE MEASURES
4.1 Apply measurement concepts and skills involving area, perimeter, and circumference in problem-solving situations.	4.1.1 Calculate both the area and the circumference of a circle with radius 2 cm. Are the anwers the same? Explain.
	4.1.2 On a clear night the beam from a lighthouse can be seen for 30 mi in all directions. Over how many square miles can the beam be seen?
	4.1.3 John has the pieces of lumber pictured below:  Can these four pieces be glued together to form a board with an area of 64 square inches?
4.2 Determine the effect on area and perimeter when changing one or two of the dimensions of a rectangle.	4.2.1 John has a board measuring 12" by 18". If he saws 3" off one side, which of the following could not be the area of the remaining board? (a) 162 in.^{2} (b) 180 in.^{2} (c) 135 in.^{2}
	4.2.2 How would doubling both the length and the width of a rectangle affect its perimeter?

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will demonstrate an understanding and use of measurement.

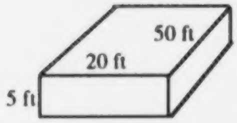
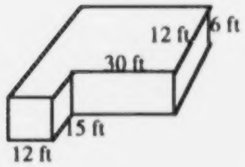
OBJECTIVES	SAMPLE MEASURES
4.3 Find the surface area of rectangular solids, pyramids, cones and cylinders.	4.3.1 If you added one inch to the length of any cube's sides, how would it affect its surface area? (a) it would increase by 1 in^2. (b) it would increase by 3 in^2. (c) it would remain the same (d) not enough information
	4.3.2 How does doubling the length of the sides of a cube affect its surface area?
4.4 Find the volume of prisms, pyramids, cylinders and cones.	4.4.1 Find the volume of a cone with diameter 1.5 inches and height 2 inches. (a) 1.5 in.^{3} (b) 1.1775 in.^{2} (c) 3.5325 in.^{2} (d) 1.1775 in.^{3}
	4.4.2 Explain the relationship between the volume of a cone and cylinder in which the radius and height are the same.
	4.4.3 How does doubling the length of the sides of a cube affect its volume?

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 4: The learner will demonstrate an understanding and use of measurement.

OBJECTIVES	SAMPLE MEASURES
4.5 Estimate solutions and solve problems related to volumes of rectangular solids.	4.5.1 Use a calculator and the fact that a cubic foot of water is approximately 7.481 gallons to determine which swimming pool pictured below requires about 37,400 gallons of water to fill.   4.5.2 List a problem from an everyday situation in which you will need to find the volume of a rectangular solid to solve the problem.

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will solve problems and reason mathematically.

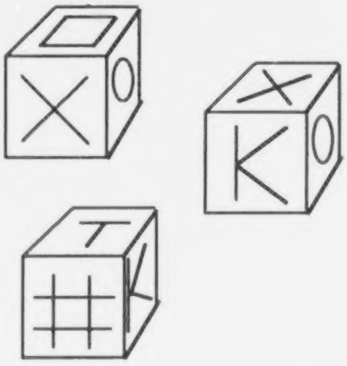
OBJECTIVES	SAMPLE MEASURES
5.1 Represent situations verbally, numerically, graphically, geometrically, or symbolically and use a variety of strategies to solve non-routine problems.	5.1.1 Fred forgot how much money he had before lunch today. He used half his money on a burger and then spent half of what was left to buy fries. He now has $1.35. How much did Fred have before lunch?
	5.1.2 A train must cross the Fritz Memorial bridge. The train is a mile long and the bridge spans only .25 miles. If the train travels at 30 miles per hour, how long will it take to completely cross the bridge?
	5.1.3 Arlene is half as old as her mother. When Arlene was 8, her mother was 30. How old is Arlene now?
5.2 Discuss alternate strategies, evaluate outcomes, and make conjectures and generalizations based on problem situations.	5.2.1 Your physics book weighs three pounds more than you English book. Your history book weighs three pounds more than your science book which weighs two pounds less than your physics book. Which weighs the most? Which weighs the least?
	5.2.2 Opal scored four three-point baskets in a basketball game. In the next game she scored two three-point baskets, five two-point baskets, and three one-point baskets. Opal said she scored a total of 29 points in the two games. Is this correct? Explain.

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 5: The learner will solve problems and reason mathematically.

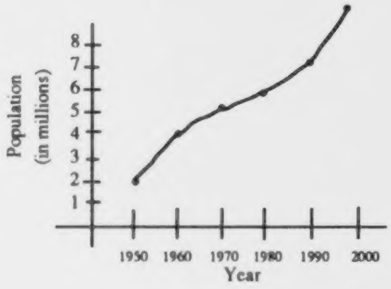
OBJECTIVES	SAMPLE MEASURES
5.3 Use concrete or pictoral models to solve problems.	5.3.1 A large bucket is filled with 8 quarts of water. You have an empty 3-quart bucket and an empty 5-quart bucket. Describe how you might use these three containers to divide the water into two equal portions.
	5.3.2 Bob lives eight blocks due east of Helen. Helen lives three blocks due west of Ilene. Where does Ilene live in relation to Bob?
	5.3.3 It is 85 miles from Greenville to Red Hill. The distance from Red Hill to Arktown is 40 miles. If Arktown is on the road from Greenville to Red Hill, how far is it from Greenville to Arktown?
	5.3.4 Look at these three cubes. What figure is on the side opposite the "X"? 

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will solve problems and reason mathematically.

OBJECTIVES	SAMPLE MEASURES
5.4 Solve problems involving the interpretation of graphs including inferences and conjectures.	5.4.1 Determine the population increase between 1970 and 1980 and the population increase between 1980 and 1990. Then estimate the population in the year 2000. 

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 5: The learner will solve problems and reason mathematically.

OBJECTIVES	SAMPLE MEASURES
5.5 Make and evaluate conjectures and arguments, using deductive and inductive reasoning.	5.5.1 All tonks are bonks. All bonks are pronks. There are 25 tonks. There are 76 pronks. Thirty three pronks are not bonks. How many bonks are not tonks?
	5.5.2 Given the sequence 5, 8, 11, . . . , what is the next number in the sequence? What rule did you use to find this number?
5.6 Investigate open-ended problems, formulate questions, and extend problem-solving situations.	5.6.1 Ed is covering a wall with decorator bricks. The wall is ten feet long and eight feet wide. Each row contains 15 bricks, and it takes three rows of bricks to cover one foot of the wall. Ed bought the bricks in boxes containing 2 dozen each. Each box cost $8.95 without tax. How much will it cost him to complete the wall?
	5.6.2 The answer to a problem is $32. Write a story problem that gives this result.

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will solve problems and reason mathematically.

OBJECTIVES	SAMPLE MEASURES
5.7 Use proportional reasoning to solve problems.	5.7.1 The number of seniors who eat lunch in the school cafeteria is about three-fourths the number of freshmen who eat lunch there. If there are 60 more freshmen than seniors eating in the cafeteria, how many seniors eat lunch there?
	5.7.2 An 18-ounce box of mixed nuts contains peanuts, cashews, and pecans. The weight of these are in a ratio of 3:1:2 respectively. How many ounces of each type are in the box?
	5.7.3 When 40 employees of a company were questioned, 24 said they had worked at the company for at least 10 years. About how many of the 200 employees are expected to have worked at the company for at least 10 years?

revised, 1992

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 6: The learner will demonstrate an understanding and use of probability and statistics.

OBJECTIVES	SAMPLE MEASURES

6.1 Create, compare, and evaluate different graphic representations of the same data.

6.1.1 A single, six-sided die was rolled 100 times. The following chart represents the outcomes.

Outcome	1	2	3	4	5	6
Frequency	16	8	12	28	20	16

Draw both a line graph and a bar graph to represent the data.

6.1.2 Evaluate the two graphs and determine which graph best represents the data and why.

6.2 Use measures of central tendency (mean, mode, and median) and range to describe meaningful data; compare two sets of unequal data.

6.2.1 A senior guard on Hillside's basketball team has scored the following points over the past nine games: 38, 18, 1, 11, 7, 7, 14, 14, and 7. Find the mean, mode, and median of his scores. Which measure best describes his "average"? Should we consider the range values? Why?

6.2.2 Use the chart below to find the mean, median mode and range for the two companies' salaries. Determine which company you would rather work for and explain why.

Salary	Company A Frequency	Company B Frequency
$ 10,000	9	
$ 20,000	2	8
$ 30,000	2	2
$ 40,000		2
$100,000	1	

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will demonstrate an understanding and use of probability and statistics.

OBJECTIVES	SAMPLE MEASURES
6.3 Evaluate appropriate uses of different measures of central tendency in problem-solving situations.	6.3.1 Roach-A-Way had developed a new chemical to destroy roaches. Ten tests were performed with the following results: Percent of roaches destroyed: 99, 99, 99, 91, 90, 88, 71, 70, 69, 60 (a) If you were the owner of the company, which measure of central tendancy would you use for advertising? Why? (b) If you were working for the pesticide commission, which measure of central tendancy would be the best for the public to use in evaluating the product? Why?
	6.3.2 At Surf and Turf, eight employees earn \$15,000 each. The assistant manager earns \$40,000, and the manager earns \$69,000. The manager said the average salary is \$23,000. Is the manager using an appropriate statistic to describe the data? Explain your answer.

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

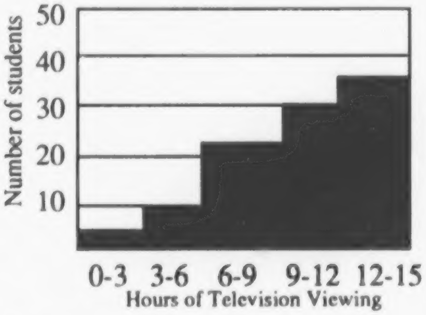
COMPETENCY GOAL 6: The learner will demonstrate an understanding and use of probability and statistics.

OBJECTIVES	SAMPLE MEASURES
6.4 Draw inferences and construct convincing arguments based on an analysis of data.	6.4.1 Thirty students took a 10-point quiz. Their scores were used to compile the following frequency table. How many students scored a passing grade if passing is a score of 7 or more?

Score	Frequency
3	1
4	0
5	4
6	3
7	2
8	5
9	12
10	3

(a) 2 (b) 22 (c) 20 (d) none of these

6.4.2 The following histogram shows the weekly television viewing times of 100 students. How many students watch television between 9 and 12 hours each week?



(a) 40 (b) 30 (c) 35 (d) 25

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will demonstrate an understanding and use of probability and statistics.

OBJECTIVES	SAMPLE MEASURES
6.5 Investigate and recognize misuses of statistical or numerical data.	6.5.1 A local jeweler began a sale on Monday. It was 70% off the ticket price. The following Friday he advertised an additional 35% off the original ticket price. Do you find this hard to believe? Explain. 6.5.2 Last week Robin did 30 sit-ups in one minute. Today, she did 45 in one minute. Her coach told her she had improved 150%. Is this correct? Explain. 6.5.3 On last week's English final, Sarah scored a 95 and was told she had the highest grade in the class. The class "average" was 80. Sarah claimed she scored the highest by a large margin. Comment on her statement. 6.5.4 Hobby Lobby advertised a 150% discount on all art supplies. Explain why you know their ad is incorrect.

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 6: The learner will demonstrate an understanding and use of probability and statistics.

OBJECTIVES	SAMPLE MEASURES
6.6 Find the mathematical and experimental probability of simple and compound events using experiments, random number generation, computer simulation, and theoretical methods.	6.6.1 In groups of two, roll two dice 100 times. Record the number of sevens and the number of elevens that occur. Find the mathematical probability of rolling a 7 and also the probability of rolling an 11. Compare with the experimental probability and explain any deviations. Why does P(7) differ from P(11)? 6.6.2 Construct a "fair" game that asks the player to roll sevens and elevens. 6.6.3 Wiley tossed a paper cup 150 times. It landed up 84 times, down 27 times, and on its side 39 times. Find each experimental probability. (a) *P* (up) (b) *P* (down) (c) *P* (side)

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will demonstrate an understanding and use of probability and statistics.

OBJECTIVES	SAMPLE MEASURES
6.7 Collect data involving two variables and display them on a scatter plot.	6.7.1 Have the students collect data involving a person's age and present income from at least 10 people and display this on a scatter plot.
	6.7.2 Use an almanac to find the area and the average depth of the world's ten largest bodies of water. Draw a scatter plot to show the relationship between these two sets of data.
6.8 Interpret data displayed on a scatter plot.	6.8.1 What can be determined from the above scatter plot (6.7.1) about age and income? Explain why.
	6.8.2 What can be determined from the scatter plot for 6.7.2 about area and depth?

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 6: The learner will demonstrate an understanding and use of probability and statistics.

OBJECTIVES	SAMPLE MEASURES
6.9 Use mathematical probability and experimental results for making predictions and decisions.	6.9.1 A bag contain 12 marbles. Six marbles are blue, four marbles are red, one marble is white, and one marble is pink. Each member of a classroom is to draw one marble from the bag, record its color, and replace it in the bag. Repeat this experiment until at least 25 events have occured. Using these results, predict the number of blue, red, white, and pink marbles that would be drawn in 100 tries. Does this seem realistic? How do these numbers differ from the mathematical expectations?
	6.9.2 Have each member of a class roll a pair of dice 20 times and record the sum of the dots on the top of the dice on each roll. Use these results to predict the number of times each sum would occur if the dice were rolled 3600 times. How do these numbers differ from the mathematical expectations?

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will use fractions, decimals, percents, and integers to solve problems.

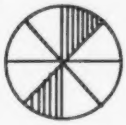
OBJECTIVES	SAMPLE MEASURES
7.1 Use operations with whole numbers, decimals, fractions and integers to solve problems demonstrating appropriate use of mental math, computation, and calculators.	7.1.1 Find the value of the following expression: $20 - 3.2 \cdot 3\frac{1}{2} + (-18)$ (a) 9.2 (b) 9.2 (c) 40.8 (d) –40.8
	7.1.2 Find the value of the following expression: $30 \cdot (-2) + 12 \div 2 - 5$
	7.1.3 Karen opened a checking acount by depositing $500. She wrote checks for $13.85, $28.14, and $230.18. She made a deposit which was a tenth of the opening balance. What is the balance in her account?
7.2 Select appropriate operations and strategies for solving a variety of applied problems involving fractions, decimals, and percents and justify the selection, using calculators, mental math, and estimates.	7.2.1 If Randy saves $350 he can buy a mountain bicycle. He has already saved $210. He figured that if he saves $32.25 a week from his car washing job for the next four weeks he will have enough money. Is Randy correct? Justify your answer by providing your strategies and methods of solving.
	7.2.2 Explain why a 20% increase followed by a 20% decrease is always greater than the original amount.
	7.2.3 A rope is cut in half and one-half is used. When one-fourth of the remaining rope is cut off and used, the piece left is 16 ft. long. How long was the rope originally? Explain your method and strategy for solving.

MATHEMATICS

Pre-Algebra

Grades Level: 9-12

COMPETENCY GOAL 7: The learner will use fractions, decimals, percents, and integers to solve problems.

OBJECTIVES	SAMPLE MEASURES
7.3 Estimate the answer to and solve problems using ratio, proportion, and percent.	7.3.1 It took 2.5 tons of fertilizer to fertilize 75 acres. How many tons of fertilizer would it take to fertilize 450 acres? (a) 6 tons (b) 15 tons (c) 187.5 tons (d) 8100 tons
	7.3.2 If a cookie recipe makes 5 dozen cookies, use proportions to determine the number by which each ingredient should be multiplied in order to make 90 cookies.
	7.3.3 Fred is leaving a 15% tip in a restaurant. A quick method to estimate 15% is to find 10% of the amount and then add one-half of the 10% amount. If the check totals $23.85, what tip should he leave? Why is this method appropriate for calculating a percent mentally?
	7.3.4 A clothing store offers a 25% discount on all shoes. How much would a pair of shoes which regularly sells for $48 cost during the sale? (a) $12 (b) $36 (c) 30 (d) none of these
	7.3.5 Estimate 24% of 16.
	7.3.6 Estimate what percent of the circle is marked. 

MATHEMATICS

Pre-Algebra

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will use fractions, decimals, percents, and integers to solve problems.

OBJECTIVES	SAMPLE MEASURES
7.4 Use ratio, proportion, and percent to solve real-world problems.	7.4.1 The tax on a car costing $9,200 is $368. At the same tax rate, what will the tax be on a car costing $12,500? (a) $437.50 (b) $562.50 (c) $250 (d) $500
	7.4.2 The ratio of weight on Earth to weight on the moon is 6:1. If you weigh 165 pounds on Earth, how much would you weigh on the moon?
	7.4.3 An insurance agent receives a 4% commission on life insurance annuities. What is the commission on a $15,000 annuity?
	7.4.4 Last year, Central High School had 800 students. This year's enrollment dropped to 600. What was the percent of decrease? (a) 75% (b) $33\frac{1}{3}\%$ (c) 25% (d) $133\frac{1}{3}\%$
	7.4.5 Before Randy started lifting weights, he weighed 100 pounds. Now, less than two years later, he weighs 125 pounds. Find the percent of increase in his weight.

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will use the language of Algebra.

OBJECTIVES		SAMPLE MEASURES	
1.1	Simplify numerical expressions.	1.1.1	Simplify: $154 + 2(5 + 3)$ (a) 165 (b) 167 (c) 170 (d) 1248
		1.1.2	Simplify: $49 - 2(16 - 11)$
1.2	Use "order of operations" to simplify numerical expressions.	1.2.1	Simplify: $4 + 4 \times 4 - 4$ (a) 4 (b) 16 (c) 28 (d) 60
		1.2.2	Write an expression involving addition and multiplication in which you would add first.
1.3	Use grouping symbols to indicate order of operations.	1.3.1	$\frac{3(2+1)}{9} - \frac{7-(4+2)}{5-4}$ (a) - 51 (b) 13 (c) 1 (d) 0
		1.3.2	Explain how to enter the following expression on a calculator and find its value: $\frac{248+467}{761-332}$
1.4	Evaluate variable expressions.	1.4.1	If $x = 5$, then $2x + 7 = ?$ (a) 10 (b) 14 (c) 17 d) 24
		1.4.2	Evaluate $a \div bc$ when $a = 24$, $b = 4$, and $c = 3$.
		1.4.3	Use two or more of the variables above and their assigned values to write an expression whose value is 8.

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will use the language of Algebra.

OBJECTIVES	SAMPLE MEASURES
1.5 Simplify exponential expressions.	1.5.1 Which of the following has the smallest value? (a) $3^2 - 2^3$ (b) $3^3 - 4^2$ (c) $(3-2)^5$ (d) $4^2 - 2^4$ 1.5.2 If $r = 2$, then $(3r)^2 = ?$ 1.5.3 If $x = 1$, then $3x - 3x^2 + 6 = ?$ 1.5.4 Use a calculator to simplify: $12 \text{ x } 4^3 - (15 \div 5)^5$.
1.6 Write expressions in exponential form.	1.6.1 Use exponents to write: 2x • 2x • 2x (a) $6x$ (b) $2(x^3)$ (c) 2^3x (d) $(2x)^3$ 1.6.2 Use exponents to rewrite 125 as a power of 5.
1.7 Translate word phrases into variable expressions.	1.7.1 Bernie was y years old last year. Which expression represents Bernie's age 3 years from now? (a) $(y+1)+3$ (b) $(y-1)+3$ (c) $y+3$ (d) $y-4$ 1.7.2 Write an expression for the cost of 4 computers and 3 printers if computers cost m dollars each and printers cost n dollars each.
1.8 Translate variable expressions and equations into word phrases.	1.8.1 Translate the expression $\frac{m+n}{2}$ into words. 1.8.2 Explain the difference between $x-3$ and $3-x$.

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will identify and apply properties of real numbers.

OBJECTIVES	SAMPLE MEASURES
2.1 Identify and apply the properties of zero for both addition and multiplication.	2.1.1 $189 + 0 = ?$ (a) 0 (b) 189 (c) –189 (d) None of these 2.1.2 $3(\frac{7}{8})(5\frac{1}{2})(0)(4) = ?$ (a) 0 (b) $17\frac{4}{3}$ (c) $\frac{420}{16}$ (d) $26\frac{1}{4}$
2.2 Identify and apply the property of one for multiplication.	2.2.1 If $\frac{2}{3}n = \frac{10}{15}$, then $n = ?$ (a) $\frac{8}{12}$ (b) $\frac{3}{2}$ (c) $\frac{5}{5}$ (d) $\frac{2}{5}$ 2.2.2 Explain how the number 1 for multiplication is similar to zero for addition.
2.3 Identify and apply the commutative and associative properties of addition and multiplication to simplify expressions or computational processes.	2.3.1 $12 + 389 + 88 = ?$ (a) 389 + 100 (b) (12 + 88) + 389 (c) (88 + 12) + 389 (d) None of the above 2.3.2 Give an example involving integers which shows that addition is commutative. 2.3.3 Give an example which shows that division is not commutative. 2.3.4 $a*b$ is defined for all real numbers a and b by$a*b = a + 2b$. Is * commutative?

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will identify and apply properties of real numbers.

OBJECTIVES	SAMPLE MEASURES
2.4 Identify and apply the distributive property.	2.4.1 Simplify: 64 x 19 + 36 x 19 (a) 1900 (b) 1216 (c) 684 (d) 23,788
	2.4.2 Explain how the distributive property helps you solve the exercise above mentally.
	2.4.3 Use the distributive property to simplify $(-4n) + 11n$.
	2.4.4 Simplify: $3(2x + 5)$ (a) $5x + 8$ (b) $6x + 5$ (c) $6x + 15$ (d) $5x + 5$

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will solve linear equations and inequalities in one variable.

OBJECTIVES	SAMPLE MEASURES
3.1 Solve an equation by using the addition or subtraction property of equality.	3.1.1 Solve: $n + 3\frac{1}{2} = 8$ (a) $\{4\frac{1}{2}\}$ (b) $\{5\frac{1}{2}\}$ (c) $\{9\}$ (d) $\{11\frac{1}{2}\}$
	3.1.2 Solve: $95 = x - 215$
3.2 Solve an equation by using the multiplication or division property of equality.	3.2.1 Solve: $15n = 105$ (a) $\{1575\}$ (b) $\{120\}$ (c) $\{90\}$ (d) $\{7\}$
	3.2.2 Write an equation which could be solved by the multiplication property of equality.
3.3 Solve an equation by using more than one property of equality.	3.3.1 Solve: $2t - 1 = 19$ (a) $\{-9\}$ (b) $\{10\}$ (c) $\{-10\}$ (d) $\{9\}$
	3.3.2 Explain the procedure you would use to solve $4x + 3 = 31$.
3.4 Solve an equation graphically.	3.4.1 Use an automatic grapher to solve $11x - 32 = 17$ to the nearest tenth. (a) $\{4.5\}$ (b) $\{1.5\}$ (c) $\{-1.1\}$ (d) $\{-1.45\}$
	3.4.2 Use an automatic grapher to solve $1.5x + 7.2 = 10$.

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will solve linear equations and inequalities in one variable.

OBJECTIVES	SAMPLE MEASURES
3.5 Solve an equation by using the distributive property.	3.5.1 Solve: $3(x + 1) = 12$ (a) $\{3\}$ (b) $\{4\}$ (c) $\{6\}$ (d) $\{8\}$ 3.5.2 Solve: $2(x - 3) + 5 = 7$
3.6 Solve an equation which contains similar terms.	3.6.1 Solve: $7x - 4x - 3 = 24$ (a) $\{9\}$ (b) $\{-9\}$ (c) $\{7\}$ (d) $\{-7\}$ 3.6.2 Explain why -3 is not a solution to the equation $6x + 2x - 3 = 21$.
3.7 Find the solution set for a linear inequality when replacement values are given for the variable.	3.7.1 Solve $2x < 4$ if $x \in \{-3, -2, -1, 0, 1, 2, 3\}$. (a) $\{-3, -2, -1, 0, 1, 2\}$ (b) $\{-3, -2, -1, 0, 1\}$ (c) $\{2\}$ (d) $\{-2\}$ 3.7.2 Solve $3x - 1 < 2$ if $x \in \{-2, -1, 0, 1, 2\}$.
3.8 Solve a linear inequality by using transformations.	3.8.1 Solve: $4x + 1 \geq 2x - 5$ (a) $x > -3$ (b) $x \geq 3$ (c) $x \leq 3$ (d) $x \geq -3$ 3.8.2 Suzie got $x \leq -2$ for the answer to $3x + 2 \leq 4x$. Identify her error.

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will solve linear equations and inequalities in one variable.

OBJECTIVES	SAMPLE MEASURES
3.9 Use a linear equation to solve problems.	3.9.1 The 25 members of Mr. Joyner's biology class went on a field trip to the zoo. There are 3 more boys than girls in his class. How many girls are there? (a) 11 (b) 14 (c) 15 (d) 10
	3.9.2 Six honeydews and three cantaloupes have 750 calories. Find the number of calories in a honeydew and in a cantaloupe if a honeydew has 20 more calories than a cantaloupe.
3.10 Use formulas to solve problems.	3.10.1 The formula for the perimeter of a rectangle of length ℓ and width w is $P = 2(\ell + w)$. A rectangle is 27 feet long and 16 feet wide. What is its perimeter? (a) 43 ft (b) 70 ft (c) 86 ft (d) 432 ft
	3.10.2 Hank used the formula $D = rt$ to find the distance he could drive if he averaged 54 mi/h for 5h. He got 59 mi for his answer. Explain his error.
	3.10.3 Tim's father is a brick mason. To estimate the number of bricks, N, needed in a wall, he uses the formula: $N = 7\,lh$, where l represents the length and h represents the height of the wall. How many bricks would be needed for a wall 4 feet high and 12 feet long?

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will solve linear equations and inequalities in one variable.

OBJECTIVES	SAMPLE MEASURES
3.11 Use inequalities to solve problems.	3.11.1 Quinn has scored 18, 17, and 30 points in his last three games. How many points must he score in the next game so that his four game average does not fall below 22 points? (a) 19 (b) 21 (c) 23 (d) 27 3.11.2 The Math Club is selling tee shirts. They bought 225 shirts and paid $8 for each shirt. If they sell the shirts for $12 each, how many must they sell before they make a profit? How many must they sell if they want to earn a profit of $1000?

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will use ratios, proportions, and percents to solve problems.

OBJECTIVES	SAMPLE MEASURES
4.1 Write a ratio in lowest terms.	4.1.1 Write as a ratio in lowest terms: 8 in. to 4 ft. (a) 1 to 6 (b) 2 to 1 (c) 1 to 5 (d) 6 to 1
	4.1.2 Write as a ratio in lowest terms: 6 months to 24 months.
	4.1.3 Find the ratio of (a) the perimeters and (b) the areas of a rectangle with sides 10m and 8m and another with sides of 15m and 12m.
4.2 Solve a proportion.	4.2.1 Solve: $\frac{x}{8} = \frac{54}{16}$ (a) {27} (b) {48} (c) {108} (d) { 432}
	4.2.2 Describe how to use a calculator to solve $\frac{15}{40} = \frac{12}{d}$.

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will use ratios, proportions, and percents to solve problems.

OBJECTIVES	SAMPLE MEASURES
4.3 Use a ratio or a proportion to solve problems.	4.3.1 An isosceles triangle has two sides of length 21 cm and a base of 24 cm. The base of a similar triangle is 16 cm. Find the perimeter of the smaller triangle. (a) 28 cm (b) 63 cm (c) 44 cm (d) 66 cm
	4.3.2 Fran and Betty work together. If their earnings are in the ratio 4 to 3 respectively, how much does Fran earn when Betty earns $650?
	4.3.3 The sales tax on a new car costing $10,500 is $420. If the sales tax rate is the same, what is the sales tax on a new car costing $16,125?
4.4 Solve problems involving percents.	4.4.1 Art paid $9 sales tax on a purchase of $150. What was the sales tax rate? (a) 5% (b) 6% (c) 16 2/3 % (d) 20%
	4.4.2 A realtor earns a 4.5% commission on sales. Find the commission on the sale of a house whose selling price is $120,000.
	4.4.3 Video City reduced the price of a video tape from $6.00 to $4.50. Explain how to use a calculator to find the percent of decrease.

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will perform operations with real numbers.

OBJECTIVES		SAMPLE MEASURES	
5.1	Determine the additive or multiplicative inverse of a number.	5.1.1	Find the additive inverse (opposite) of −7. (a) −7 (b) 7 (c) $\frac{1}{7}$ (d) $-\frac{1}{7}$
		5.1.2	Give an example of a number and its multiplicative inverse (reciprocal).
5.2	Distinguish between rational and irrational numbers.	5.2.1	Which of the following is an irrational number? (a) $\sqrt{169}$ (b) $\sqrt{1024}$ (c) $-\sqrt{16}$ (d) $\sqrt{7}$
		5.2.2	Write a rational number between $\frac{1}{2}$ and $\frac{3}{4}$.
5.3	Determine the absolute value of expressions.	5.3.1	If $x = 1$, $\lvert x - 6 \rvert =$ _____? (a) −5 (b) 5 (c) 7 (d) −7
		5.3.2	Simplify: $\lvert 7 - 3 \rvert + \lvert 3 - 7 \rvert$

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will perform operations with real numbers.

OBJECTIVES	SAMPLE MEASURES
5.4 Compare real number expressions.	5.4.1 Replace the ? with <, =, or > to make a true statement: $\sqrt{81} + \sqrt{64} + \sqrt{49} \ \underline{?}\ \sqrt{81+64+49}$ (a) < (b) > (c) = (d) none of the above
	5.4.2 Use – 8, 2, 5 and two inequality symbols to write a true statement.
	5.4.3 Write a number under a radical sign which is between 1 and 5.
5.5 Simplify variable expressions involving addition and subtraction.	5.5.1 Simplify: $\frac{3x}{x-2} + \frac{6}{2-x}$ (a) $\frac{3x+2}{x-2}$ (b) $\frac{3x-6}{2-x}$ (c) –3 (d) 3
	5.5.2 Simplify: $14x^2 - 6x + 8 - 6x^2 + x - 11$
5.6 Simplify variable expressions involving multiplication and division.	5.6.1 Simplify: $(-\frac{1}{3}a)(-3a)(-a^2)$ (a) $9a^3$ (b) a^3 (c) $-a^3$ (d) $-a^4$
	5.6.2 Simplify: $102x^2 + 6x$

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will perform operations with real numbers.

OBJECTIVES		SAMPLE MEASURES	
5.7	Simplify real number expressions with or without a calculator.	5.7.1	Simplify: $\frac{264 + 6^2}{12}$ (a) 22 (b) 23 (c) 25 (d) 35
		5.7.2	Explain the sequence of keystrokes you would use on your calculator to simplify: $\frac{3}{5}(2^5 - 3^2)$
5.8	Simplify radicals.	5.8.1	Simplify: $\sqrt{75}$ (a) $5\sqrt{3}$ (b) $3\sqrt{5}$ (c) $3\sqrt{25}$ (d) $25\sqrt{3}$
		5.8.2	Simplify: $\sqrt{192}$
5.9	Multiply or divide two radical expressions.	5.9.1	Simplify: $\sqrt{6} \cdot \sqrt{12}$ (a) $6\sqrt{2}$ (b) 72 (c) $\sqrt{72}$ (d) $\sqrt{612}$
		5.9.2	Simplify: $\sqrt{48} \div \sqrt{3}$

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will perform operations with real numbers.

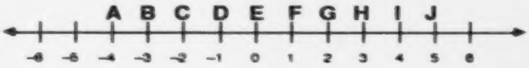
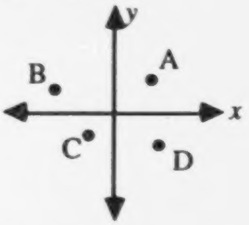
OBJECTIVES	SAMPLE MEASURES
5.10 Use a calculator to find the approximate square root of a number.	5.10.1 Use your calculator to find $\sqrt{1321}$ to the nearest hundredth. (a) 36.33 (b) 3.63 (c) 36.03 (d) 30.36
	5.10.2 Between which two integers does $\sqrt{29}$ lie?
5.11 Add or subtract radical expressions.	5.11.1 Simplify: $10\sqrt{6} - 3\sqrt{6} + \sqrt{6}$ (a) $14\sqrt{6}$ (b) $9\sqrt{6}$ (c) $7\sqrt{6}$ (d) $8\sqrt{6}$
	5.11.2 Simplify: $5\sqrt{2} + 3\sqrt{6} + 2\sqrt{8} - 5\sqrt{54}$
5.12 Use the Pythagorean Theorem to solve problems.	5.12.1 The length of a rectangle is 10 cm. Its width is 8 cm. Use your calculator to find the length of a diagonal. (a) 12.8 cm (b) 25.6 cm (c) 57.9 cm (d) 22.8 cm
	5.12.2 The length of one side of a right triangle is 24 cm. The length of the hypotenuse is 25 cm. Write and solve an equation to find the unknown side.

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will demonstrate an elementary understanding of relations and functions.

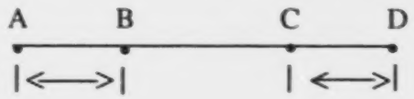
OBJECTIVES	SAMPLE MEASURES
6.1 Graph and locate sets of real numbers on a number line.	List the letter for the point whose coordinate is given:  6.1.1 –2 (a) C (b) G (c) D (d) F 6.1.2 Locate $-\frac{1}{2}$ on the number line above and label this point *M*.
6.2 Graph ordered pairs of numbers on the coordinate plane and interpret information related to these sets of points.	6.2.1 Which point is most likely to be the graph of (4, –1)?  6.2.2 Suppose the *x*– coordinate of an ordered pair is negative. In which quadrant(s) might the point be graphed?

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will demonstrate an elementary understanding of relations and functions.

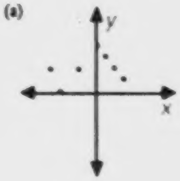
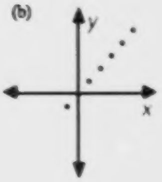
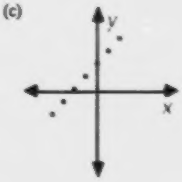
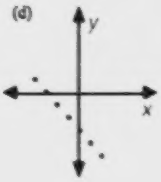
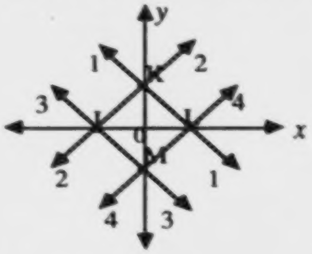
OBJECTIVES	SAMPLE MEASURES
6.3 Find the distance between two points on a number line.	6.3.1 On a number line, if point *M* has coordinate –3 and point *N* has coordinate 4, how long is *MN*? (a) 7 (b) –7 (c) 1 (d) –1 6.3.2  In the figure above, if segment AC has length 10, how long is segment BD?
6.4.1 Graph a relation on the coordinate plane.	6.4.1 What is the graph of the relation: *A* = {(4, 1), (0, 0), (–4, –1)} (a) a point (b) three points on the coordinate plane (c) a line segment (d) three line segments

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will demonstrate an elementary understanding of relations and functions.

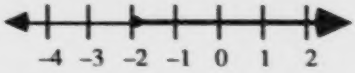
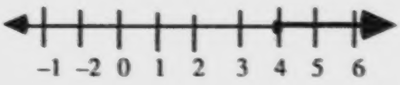
OBJECTIVES	SAMPLE MEASURES
6.5 Graph a relation given an equation and a domain.	6.5.1 Which is the graph of $y = x + 2$ when $x \in \{-3, -2, -1, 0, 1, 2, 3\}$?     6.5.2 Draw the graph of $y = x + 2$ when $x \in \{\text{real numbers}\}$. Explain how this differs from the graph in 6.5.1
6.6 Identify intercepts as numbers.	6.6.1 Which line has an x–intercept of of –2 and a y–intercept of 1?  (a) 1 (b) 2 (c) 3 (d) 4 6.6.2 Use the graph to determine which line has an x–intercept of 2 and a y– intercept of 1.

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will solve and graph linear equations and inequalities.

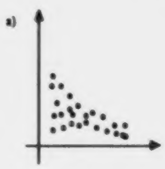
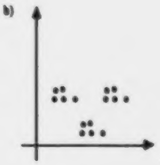
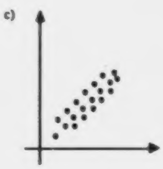
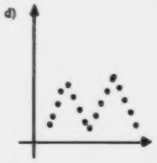
OBJECTIVES	SAMPLE MEASURES
7.1 Graph an inequality on a number line.	7.1.1 The graph on the number line below is:  (a) –2, –1, 0, 1, 2 (b) $n < 4$ (c) $n \geq -2$ (d) $n > -2$ 7.1.2 Draw the graph of $n < 13$.
7.2 Solve an inequality and graph its solution set.	7.2.1  This graph is the solution set of: (a) $-4w + 2 \leq -14$ (b) $-4w + 2 \geq -14$ (c) $w > 4$ (d) $w > 5$ 7.2.2 Solve and graph the solution set of $4 + x \leq 1$.

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will solve and graph linear equations and inequalities.

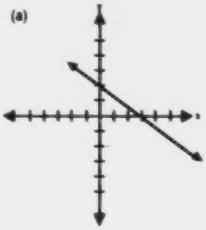
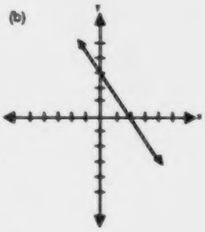
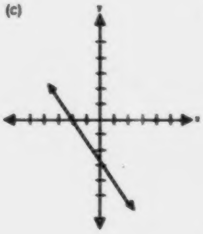
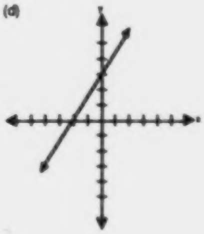
OBJECTIVES	SAMPLE MEASURES
7.3 Determine if data are behaving in a linear fashion.	7.3.1 Which data appear linear?    
	7.3.2 Describe a relationship between two variables that you believe would behave in a linear fashion.
7.4 Find the solution set of open sentences in two variables when given replacement sets for the variables.	7.4.1 Which ordered pair lies on the graph of the line $2x + 5y = 4$? (a) $(-3, 2)$ (b) $(3, -2)$ (c) $(\frac{1}{2}, -1)$ (d) $(-\frac{2}{5}, 0)$
	7.4.2 Show why $(-2, -1)$ is not a solution of $x - 3y = -1$.

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will solve and graph linear equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
7.5 Graph a linear equation in two variables.	7.5.1 Which is the graph of $2x + 3y = 6$? (a)  (b)  (c)  (d)  7.5.2 Draw the graph of $x - 2y = 4$. 7.5.3 Use a computer or graphics calculator to display $2x - y = -8$.

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will solve linear equations with two variables.

OBJECTIVES	SAMPLE MEASURES
8.1 Solve systems of linear equations.	8.1.1 Use graphs to determine the solution of the following system of equations: $2x - y = 1$ $x + y = 5$ (a) (1, 4) (b) (0, –1) (c) (2, 3) (d) (3, 2)
	8.1.2 Solve by graphing: $y - 2x = 5$ $y - x = -3$
8.2 Use substitution to solve systems of linear equations.	8.2.1 Solve this system by the substitution method: $n = 4m$ $m - n = 9$ (a) (–3, –12) (b) (8, 1) (c) (3, 12) (d) (12, 3)
	8.2.2. Solve: $4x + 5y = -1$ $3x - y = 23$

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will solve linear equations with two variables.

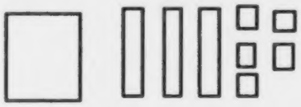
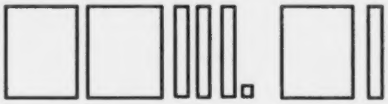
OBJECTIVES	SAMPLE MEASURES
8.3 Identify inconsistent systems.	8.3.1 How many solutions does this system have? $3x + 4y = 1$ $6x + 8y = 2$ (a) 0 solutions (b) more than 2 solutions (c) 1 solution (d) 2 solutions 8.3.2 Solve: $2x - 3y = -12$ $4x - 6y = 20$
8.4 Use a computer or graphics calculator to solve systems of linear equations.	8.4.1 Use a computer or graphics calculator to find the solution of this system: $5x - 3y = 25$ $6x + 5y = 30$ (a) (5, 0) (b) (0, 5) (c) $(0, 8\frac{1}{3})$ (d) (0, 6) 8.4.2 Use a computer or graphics calculator to find the solution of this system to the nearest tenth: $7x + 4y = 12$ $2x - 3y = 1$

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will perform operations with polynomials.

OBJECTIVES	SAMPLE MEASURES
*9.1 Use models to represent polynomials.	9.1.1 Write a polynomial for the model:  (a) $x^2 + 3x + 5$ (b) $3x^2 + x + 5$ (c) $5x^2 + 3x + 1$ (d) $x^2 + 5x + 3$
	9.1.2 Draw a model to represent $2x^2 + x + 4$.
*9.2 Use models to find the sum or difference of two polynomials.	9.2.1 Write a polynomial for each model. Then find the sum of the two polynomials.  (a) $3x^2 + 4x + 1$ (b) $x^2 + 4x + 1$ (c) $x^2 + 2x + 1$ (d) $2x^2 + 2x - 1$
	9.2.2 Draw a model to represent: $(x^2 + 3x - 2) - (2x^2 - x - 1)$.

*These objectives are not included in the Algebra I curriculum. They are listed in this course since they should enable students to gain a better understanding of polynomials and some of the operations related to them.

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will perform operations with polynomials.

OBJECTIVES	SAMPLE MEASURES
9.3 Add and subtract polynomials.	9.3.1 Simplify: $(5x^2 - x + 3) + (x^2 + 3x - 7)$ (a) $6x^2 + 4x + 10$ (b) $6x^2 - 4x - 10$ (c) $5x^2 + 3x - 4$ (d) $6x^2 + 2x - 4$
	9.3.2 Simplify: $(x^4 + 4x^2 - 6x - 8) - (x^3 - x^2 + 3x + 2)$
9.4 Multiply monomials.	9.4.1 Multiply: $(3x^2y)(-5xy^2)$ (a) $-15x^2y^2$ (b) $-8x^3y^3$ (c) $-8x^2y^2$ (d) $-15x^3y^3$
	9.4.2 Multiply: $(2n^4)(5n^3)(3n)$
9.5 Find an indicated power of a monomial.	9.5.1 Simplify: $(-5a^3)^2$ (a) $25a^6$ (b) $25a^5$ (c) $-10a^6$ (d) $-10a^5$
	9.5.2 Simplify: $(-xy^2)^3$
9.6 Multiply a polynomial by a monomial.	9.6.1 Multiply: $3m^2(5m^2 - 6m + 7)$ (a) $15m^4 - 18m^2 + 21m^2$ (b) $15m^4 + 3m^2$ (c) $15m^4 - 18m^3 + 21m^2$ (d) $15m^4 - 9m^3 + 10m^2$
	9.6.2 Multiply: $-\frac{2}{5}x(10x^2 + 5x - 15)$

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will perform operations with polynomials.

OBJECTIVES	SAMPLE MEASURES
9.7 Divide two monomials.	9.7.1 Divide: $\frac{72x^6}{9x^2}$ (a) $8x^4$ (b) $8x^3$ (c) $\frac{x^4}{8}$ (d) $\frac{8}{x^4}$ 9.7.2 Divide: $\frac{64x^2y^3}{-4xy^2}$

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 10: The learner will simplify expressions with algebraic fractions.

OBJECTIVES	SAMPLE MEASURES
10.1 Simplify algebraic fractions.	10.1.1 Simplify: $\frac{a(x-1)}{ax}$ (a) 2 (b) $x-1$ (c) $\frac{x-1}{x}$ (d) a 10.1.2 Simplify: $\frac{6x(x+5)}{12x^2(x+5)}$
10.2 Use the distributive property to simplify fractions.	10.2.1 Simplify: $\frac{3x+3y}{3x-3}$ (a) $\frac{1}{-1}$ (b) $\frac{x+3}{x-3}$ (c) $\frac{1+y}{0}$ (d) $\frac{x+1}{x-1}$ 10.2.2 Simplify: $\frac{x^3+2x^2-3x}{2x^3+2x^2-4x}$ 10.2.3 Explain how the distributive property is used to simplify algebraic fractions.
*10.3 Multiply algebraic fractions.	10.3.1 Simplify: $\frac{2xy}{3x^2} \cdot \frac{9x}{5y^2}$ (a) $\frac{6}{5y^2}$ (b) $\frac{6}{5}y^2$ (c) $\frac{2x}{5y}$ (d) $\frac{2x}{5y^2}$ 10.3.2 Simplify: $\frac{(x+3)(x-2)}{(x+3)(x+4)} \cdot \frac{(x+4)(x-1)}{(x+2)(x-2)}$

*This objective is not included in the Algebra I curriculum. If time permits, it should be presented to students as a way of showing a connection between arithmetic and algebraic concepts and skills.

MATHEMATICS

Algebra I - A

GRADE LEVEL: 9-12

COMPETENCY GOAL 10: The learner will simplify expressions with algebraic fractions.

OBJECTIVES	SAMPLE MEASURES
*10.4 Divide algebraic fractions.	10.4.1 Simplify: $\frac{2}{x-1} \div \frac{-3}{x-1}$ (a) $-\frac{2}{3}$ (b) $\frac{-6}{x-1}$ (c) $-\frac{3}{2}$ (d) $\frac{(x-1)^2}{-6}$ 10.4.2 $\frac{(x+5)(x-3)}{(x+3)(x-3)} \div \frac{(x-6)(x-1)}{(x-6)(x+3)}$

*This objective is not included in the Algebra I curriculum. If time permits, it should be presented to students as a way of showing a connection between arithmetic and algebraic concepts and skills.

NOTES

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 1: The learner will perform operations with real numbers.

OBJECTIVES		SAMPLE MEASURES	
1.1	Add or subtract real numbers.	1.1.1	Add: $-51\frac{1}{2} + 19\frac{1}{2}$ (a) 70 (b) –70 (c) –32 (d) 32
		1.1.2	Write an expression involving the additon of integers which has –5 as its answer.
		1.1.3	Write an expression involving the subtraction of integers which has –8 as its answer.
1.2	Multiply or divide real numbers.	1.2.1	Multiply: $(-\frac{2}{3})(\frac{4}{9})$ (a) $-\frac{2}{3}$ (b) $-\frac{8}{27}$ (c) $\frac{8}{27}$ (d) $\frac{2}{3}$
		1.2.2	Louis multiplied three rational numbers and got –150 for his answer. How many of these numbers are negative?
		1.2.3	Write an expression involving division which has $-\frac{2}{3}$ as its answer.
1.3	Use the order of operations to simplify expressions.	1.3.1	Simplify: $(-4)(-4) - 4 + 4 + 4$ (a) 3 (b) 13 (c) 4 (d) 1
		1.3.2	Write an expression which involves subtraction and division in which you subtract first.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 1: The learner will perform operations with real numbers.

OBJECTIVES	SAMPLE MEASURES
1.4 Evaluate exponential expressions.	1.4.1 Evaluate $a^3 b^4$ when $a = -1$ and $b = 3$. (a) -12 (b) -3 (c) -81 (d) 81
	1.4.2 If $a = -5$, how does $a^2 + a$ compare with -30?
	1.4.3 Simplify: $(-1)^{1992}$ (a) -1 (b) 1 (c) -1992 (d) 1992
1.5 Evaluate polynomials.	1.5.1 Evaluate: $3x^2 - 3x - 6$ if $x = 1$ (a) 1 (b) 0 (c) -1 (d) -6
	1.5.2 Assign a value to x so that the expression $x^2 - 2x + 4$ has a value less than 0 or equal to -4.
1.6 Interpret zero as an exponent.	1.6.1 Simplify: $(-5)^0$ (a) 1 (b) 0 (c) 5 (d) -5
	1.6.2 Simplify: $(5a)^0$
1.7 Interpret negative exponents.	1.7.1 Simplify: 2^{-3} (a) -8 (b) -6 (c) $\frac{1}{8}$ (d) 8
	1.7.2 Simplify: $(4^0)^{-1}$

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 1: The learner will perform operations with real numbers.

OBJECTIVES	SAMPLE MEASURES
1.8 Write numbers in scientific notation and use this notation with the calculator.	1.8.1 Write 12,300 in scientific notation. (a) 1.23×10^{-4} (b) 12.3×10^{-3} (c) 12.3×10^{3} (d) 1.23×10^{4}
	1.8.2 Which sequence of key strokes would you use on your calculator to enter the number 2.5×10^{6}?
	1.8.3 Use your calculator to compute 853,000,000 + 4,520,000 and express your answer in scientific notation.
1.9 Simplify real number expressions with and without a calculator.	1.9.1 Simplify: $\dfrac{8^2 + 284}{16}$ (a) 21.75 (b) 81.75 (c) 288 (d) 5329
	1.9.2 Explain the sequence of keystrokes you would use on your calculator to simplify: $\frac{3}{5}(2^4 + 3^2)$.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 2: The learner will identify the properties of real numbers and apply them to numerical and algebraic expressions.

OBJECTIVES	SAMPLE MEASURES
2.1 Recognize and use properties of addition.	2.1.1 The equation $89n + (11n + 10n) = (89n + (11n) + 10n$ illustrates the: (a) addition property of zero (b) addition property of opposites (c) commutative property of addition (d) associative property of addition
	2.1.2 Using integers, write a statement which illustrates the commutative property of addition.
2.2 Recognize and use properties of multiplication.	2.2.1 The equation $55n \cdot 4 = 4 \cdot 55n$ illustrates the: (a) associative property of multiplication (b) commutative property of multiplication (c) multiplication property of one (d) multiplication property of zero
	2.2.2 Using numbers and variables, write an equation which illustrates the multiplication property of one.
2.3 Use the distributive property to simplify expressions.	2.3.1 Multiply: $-3(x - 5)$ (a) $3x - 15$ (b) $-8x$ (c) $-3x + 15$ (d) $-3x - 5$
	2.3.2 Explain how $x \cdot x + 3$ and $x(x + 3)$ are different.
	2.3.3 If $3(16) - 3 = 3R$, then $R = \underline{?}$.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 2: The learner will identify the properties of real numbers and apply them to numerical and algebraic expressions.

OBJECTIVES	SAMPLE MEASURES
2.4 Use the distributive property to factor a common monomial term in an algebraic expression.	2.4.1 Factor: $15x^2 - 3x$ (a) $3(x^2 - x)$ (b) $3x^2(5 - 3x)$ (c) $3x(5x - 1)$ (d) $3x$ 2.4.2 Factor: $20x^2 - 30x + 40$
2.5 Use number properties to combine like terms.	2.5.1 Simplify: $-4a - 6 + 10a$ (a) 0 (b) $6a - 6$ (c) $-20a^2$ (d) $240a^2$ 2.5.2 Simplify: $20a - 16a + 11 - 9a^2 + 7a - 15$

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 3: The learner will solve linear equations and inequalities with one variable and graph their solutions.

OBJECTIVES	SAMPLE MEASURES
3.1 Solve an equation by using the addition property of equality and the idea of an additive inverse.	3.1.1 Solve: $x - 10 = -6$ (a) $\{-16\}$ (b) $\{16\}$ (c) $\{-4\}$ (d) $\{4\}$
	3.1.2 Write an equation which can be solved by the addition property of equality.
	3.1.3 Explain how the addition property of equality is used to solve $x + 25 = 17$.
3.2 Solve an equation by using the multiplication property of equality and the idea of a multiplicative inverse.	3.2.1 Solve: $\frac{x}{-20} = 10$ (a) $\{-2\}$ (b) $\{2\}$ (c) $\{-200\}$ (d) $\{200\}$
	3.2.2 Write an equation which can be solved by the multiplication property of equality.
3.3 Solve an equation graphically or by using more than one property of equality.	3.3.1 Solve: $2x - 1 = 19$ (a) $\{9\}$ (b) $\{-9\}$ (c) $\{10\}$ (d) $\{-10\}$
	3.3.2 Use an automatic grapher to solve $13x - 41 = 17$ to the nearest tenth.
	3.3.3 Write an equation which involves more than one property of equality and has 7 for its solution.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 3: The learner will solve linear equations and inequalities with one variable and graph their solutions.

OBJECTIVES	SAMPLE MEASURES
3.4 Solve an equation which has a variable in both members.	3.4.1 Solve: $6x + 3 = 4x + 11$ (a) {1} (b) {4} (c) {6} (d) {1.4}
	3.4.2 Solve: $7x + 9 = 5x - 13$
3.5 Translate word statements into equations and solve them.	3.5.1 A parking lot has spaces for 500 cars. The number of spaces for compact cars is 35 more than half the spaces for standard cars. How many spaces are available for compact cars? (a) 310 (b) 155 (c) 285 (d) 190
	3.5.2 The perimeter of a rectangle is 64m. The length is 8m longer than the width. Find the dimensions.
3.6 Solve an equation in which the numerical coefficient is a fraction.	3.6.1 Solve: $\frac{3}{4}x - 12 = \frac{x}{3} - 7$ (a) {−12} (b) {12} (c) {1} (d) {−1}
	3.6.2 Solve: $\frac{2}{5}x + 12 = x$

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 3: The learner will solve linear equations and inequalities with one variable and graph their solutions.

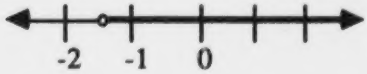
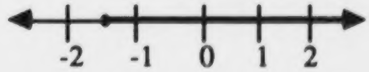
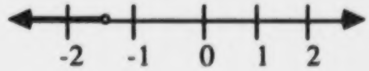
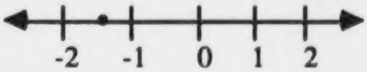
OBJECTIVES	SAMPLE MEASURES
3.7 Solve a formula for one of its variables or find the value of a variable when values of other variables are given.	3.7.1 Solve $y = mx + b$ for m. (a) $m = \frac{b+y}{x}$ (b) $m = \frac{y-b}{x}$ (c) $m = \frac{b+x}{y}$ (d) $m = \frac{b-x}{y}$
	3.7.2 The perimeter of a square can be found by the formula $P = 4s$. Rewrite this formula to solve for the length of a side (s).
	3.7.3 If a square has a perimeter of 18m, how long is each side?
3.8 Use formulas to solve problems	3.8.1 Use the formula $A = \frac{1}{2}bh$ to find the area (A) of a triangle when $b = 16\text{cm}$ and $h = 10\text{cm}$. (a) 160cm^2 (b) 40cm^2 (c) $26\frac{1}{2}\text{cm}^2$ (d) 80cm^2
	3.8.2 Phyllis used the formula above to find the area of a triangle with base 12cm and height 8cm. She got 96cm^2 for her answer. Explain her error.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 3: The learner will solve linear equations and inequalities with one variable and graph their solutions.

OBJECTIVES	SAMPLE MEASURES
3.9 Graph an inequality on a number line.	3.9.1 Which of these is the graph of $x \geq -1\frac{1}{2}$, where $x \in \{\text{real numbers}\}$. (a)  (b)  (c)  (d)  3.9.2 Draw the graph of $x \geq -1\frac{1}{2}$ where $x \in \{\text{integers}\}$.
3.10 Find the solution set of a linear inequality when replacement values are given for the variable.	3.10.1 Solve: $2x > -2$ if $x \in \{-3, -2, -1, 0, 1, 2, 3\}$. (a) $\{0, 1, 2, 3\}$ (b) $\{-1, 0, 1, 2, 3\}$ (c) Ø (d) $\{-2, -1, 0, 1, 2, 3\}$ 3.10.2 Solve: $3x - 1 < x + 7$ if $x \in \{-3, -2, -1, 0, 1, 2, 3\}$.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 3: The learner will solve linear equations and inequalities with one variable and graph their solutions.

OBJECTIVES	SAMPLE MEASURES
3.11 Solve a linear inequality by using transformations.	3.11.1 Solve: $3x - 9 \leq x + 7$ (a) $x \leq 1$ (b) $x \leq 8$ (c) $x \leq 4$ (d) $x \leq \frac{1}{2}$
	3.11.2 Solve: $\frac{2}{3}x - 7 \leq 2$
3.12 Use inequalities to solve problems.	3.12.1 Erin earns $24,000 per year in salary and 8% commission on her sales. If she must have a total income of at least $30,000 to pay her expenses, what is the least amount she must sell in order to do this? (a) $ 6,000 (b) $14,000 (c) $54,000 (d) $75,000
	3.12.2 Sal's scores on his first four tests were 85, 89, 81, and 90. What score must he make on his next test to have an average of at least 87 for all five tests?
3.13 Find the solution set of combined inequalities.	3.13.1 Solve: $2 \leq x + 1 < 5$ (a) $1 \leq x < 4$ (b) $1 \leq x < 6$ (c) $3 \leq x < 6$ (d) $3 \leq x < 4$
	3.13.2 Solve: $-5 < 1 - 2x \leq 3$

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 3: The learner will solve linear equations and inequalities with one variable and graph their solutions.

OBJECTIVES	SAMPLE MEASURES
3.14 Solve a simple equation involving absolute value.	3.14.1 Solve $\lvert x - 4 \rvert = 6$. (a) Both –2 and 10 (b) –2 only (c) 10 only (d) No real number solution 3.14.2 Explain why –3 is not a solution to $\lvert x - 4 \rvert = -7$.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 4: The learner will demonstrate an elementary understanding of relations and functions.

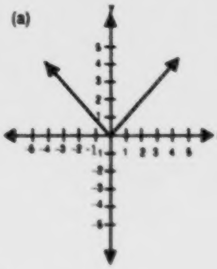
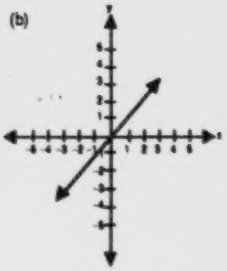
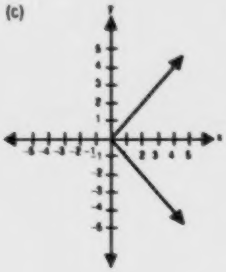
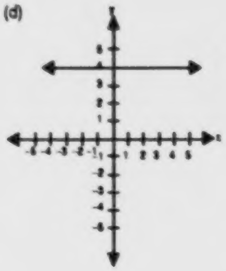
OBJECTIVES	SAMPLE MEASURES
4.1 Graph ordered pairs of numbers on the coordinate plane and interpret information related to those sets of points.	4.1.1 In which quadrant is the graph of $(2, -5)$? (a) I (b) II (c) III (d) IV
	4.1.2 Plot the points $(2, 0)$, $(0, 3)$, and $(-2, 4)$ to determine if they are collinear.
4.2 Graph a relation on the coordinate plane.	4.2.1 Which is the graph of the relation $B = \{(-4, 3), (-1, 1), (3, 2)\}$? (a) a point (b) a line segment (c) three non-collinear points (d) three line segments
	4.2.2 Graph the relation: $C = \{(-2, 1), (0, 2), (2, 3), (2, -1)\}$

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 4: The learner will demonstrate an elementary understanding of relations and functions.

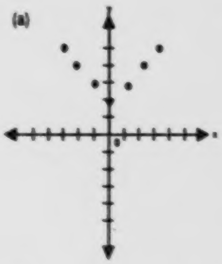
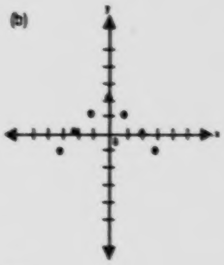
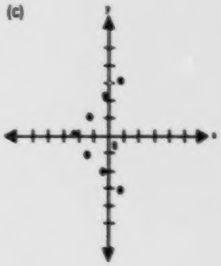
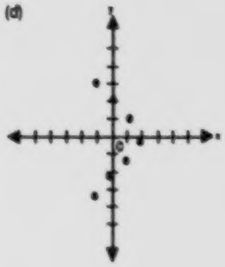
OBJECTIVES	SAMPLE MEASURES
4.3 Distinguish between a relation and a function.	4.3.1 Tell which graph is not a function. (a)  (b)  (c)  (d)  4.3.2 Give an example to illustrate the fact that the following statement is false: "Every relation is a function."

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 4: The learner will demonstrate an elementary understanding of relations and functions.

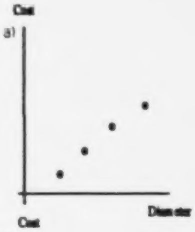
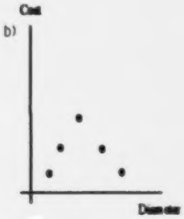
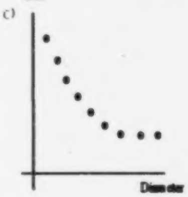
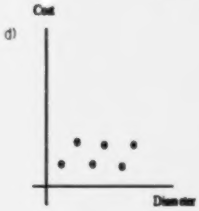
OBJECTIVES	SAMPLE MEASURES
4.4 Graph a relation given an equation and a domain.	4.4.1 Which is the graph of $y = \lvert x \rvert + 2$ when $x \in \{-3, -2, -1, 0, 1, 2, 3\}$? (a)  (b)  (c)  (d) 
	4.4.2 Draw the graph of $y = \lvert x \rvert + 1$ when $x \in \{-3, -2, -1, 0, 1, 2, 3\}$ and explain how it differs from the graph in 4.4.1.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 4: The learner will demonstrate an elementary understanding of relations and functions.

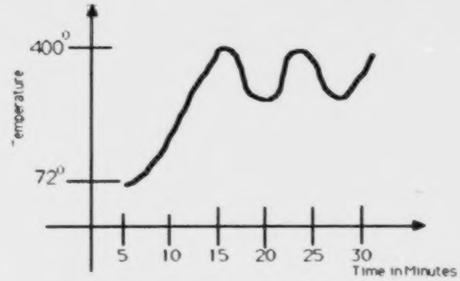
OBJECTIVES	SAMPLE MEASURES
4.5 Sketch a reasonable graph for a given relationship	4.5.1 Which graph best represents the relationship between the cost of pizzas of various diameters?     4.5.2 Sketch a reasonable graph which represents an individual's height as he ages. Label both axes and write a sentence or two to justify the shape or behavior of your graph.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 4: The learner will demonstrate an elementary understanding of relations and functions.

OBJECTIVES	SAMPLE MEASURES
4.6 Interpret a graph in a real-world setting.	4.6.1 The graph of the temperature of an oven over a 30-minute period is provided below. At about what time is the temperature 200^0 F?  (a) 5 minutes (b) 10 minutes (c) 15 minutes (d) 20 minutes 4.6.2 The graph of the temperature of an oven over a 30-minute period of time is provided above. (a) Why is the early temperature 72^0? (b) What is represented over the time period from 5 to 15 minutes? (c) What is represented over the time period from 15 to 30 minutes? (d) If the oven is turned off after 40 minutes, sketch a graph of temperature vs. time over the period of time from 40 to 100 minutes.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 4: The learner will demonstrate an elementary understanding of relations and functions.

OBJECTIVES	SAMPLE MEASURES
4.7 Use a computer or graphing calculator to explore the graphs of functions.	4.7.1 Graph $y = x$ and $y = 3x + 5$ using an automatic grapher. How many points of intersection exist? (a) 0 (b) 1 (c) 2 (d) 3 4.7.2 Graph $y = \lfloor x \rfloor$ and $y = \lfloor x \rfloor + 3$ using an automatic grapher. Describe how the two graphs differ.
4.8 Compare ordered pairs of numbers to the line $y = x$ and interpret the results.	4.8.1 Five students begin training for soccer. Prior to the training program, the students' weights were 128, 132, 154, 165 and 170 pounds. After the training program, the students' weights were 134, 140, 152, 165, and 165 respectively. How many of the ordered pairs in this data set fall on, above, or below the line $y = x$? (a) All above (b) 2 above, 2 below, 1 on (c) All on (d) 4 above, 1 on 4.8.2 Describe how you know when an ordered pair such as (5, 12) is above, on, or below the line $y = x$.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 5: The learner will simplify radical expressions which contain variables.

OBJECTIVES	SAMPLE MEASURES
5.1 Simplify square root radicals.	5.1.1 Simplify: $\sqrt{128x^3}$ (a) $2x\sqrt{64x^2}$ (b) $\sqrt{2x}\cdot\sqrt{64x^2}$ (c) $8x\sqrt{2x}$ (d) $4x\sqrt{2x}$ 5.1.2 Simplify: $-3x\sqrt{125x}$
5.2 Simplify radical expressions involving products and quotients.	5.2.1 Simplify: $\sqrt{\frac{2x}{3}}\cdot\sqrt{6x^3}$ (a) $\frac{\sqrt{12x}}{3}$ (b) $12\sqrt{3x^2}$ (c) $-2x^2$ (d) $2x^2$ 5.2.2 Simplify: $\frac{\sqrt{216x}}{\sqrt{6x}}$
5.3 Simplify the sums and differences of radical expressions.	5.3.1 Simplify: $\sqrt{32x}+\sqrt{18x}-\sqrt{128x}$ (a) $-15\sqrt{2x}$ (b) $-\sqrt{2x}$ (c) $\sqrt{2x}$ (d) $15\sqrt{2x}$ 5.3.2 Simplify: $3\sqrt{5x}-3\sqrt{20x}-\sqrt{80x}$

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 5: The learner will simplify radical expressions which contain variables.

OBJECTIVES	SAMPLE MEASURES
5.4 Find the approximate square root of a number with and without a calculator.	5.4.1 $\sqrt{27}$ is between: (a) 4 and 5 (b) 6 and 7 (c) 5 and 6 (d) 3 and 4 5.4.2 Use a calculator to find $\sqrt{127}$ to the nearest hundredth.
5.5 Compare real number expressions.	5.5.1 Replace the ? with <, =, or > to make a true statement: $\sqrt{100} - \sqrt{64} \ \underline{?} \ \sqrt{100-64}$ (a) = (b) < (c) > (d) none of these 5.5.2 Give an example to show when $n < \frac{1}{n}$ is a true statement. Then give an example to show when it is false.
5.6 Multiply two binomials which contain square roots.	5.6.1 Multiply: $(\sqrt{3} + 1)(\sqrt{3} - 1)$ (a) -2 (b) $2\sqrt{3}$ (c) $-2\sqrt{3}$ (d) 2 5.6.2 Multiply: $(\sqrt{5} - 2)^2$

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 5: The learner will simplify radical expressions which contain variables.

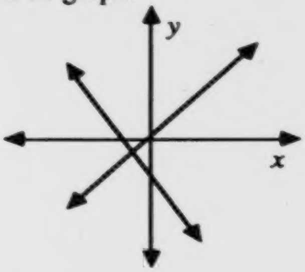
OBJECTIVES	SAMPLE MEASURES
5.7 Solve equations which contain radicals.	5.7.1 Solve: $3\sqrt{x-2} = x$ Check your solution: (a) Ø (b) $\{3\}$ (c) $\{6\}$ (d) $\{3, 6\}$ 5.7.3 Explain why it is important to check your answer when you solve an equation which contains a radical.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 6: The learner will graph and solve systems of linear equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
6.1 Solve a system of two linear equations in two variables by graphing.	6.1.1 Write the solution set of the system from its graph.  (a) $(1, -1)$ (b) $(-1, 1)$ (c) $(1, -1)$ (d) $(-1, -1)$
	6.1.2 Solve by graphing: $x - 2y = 0$ $x + y = 3$
6.2 Solve a system of two linear equations by the substitution method.	6.2.1 Solve by the substitution method: $y = 1 - x$ $x - y = 3$ (a) $(2, -1)$ (b) $(-1, 2)$ (c) $(-2, 1)$ (d) $(-2, -1)$
	6.2.2 Solve by the substitution method: $x = 2y - 1$ $3x - y = 4$

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 6: The learner will graph and solve systems of linear equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
6.3 Use the addition or subtraction method to find the solution to a pair of linear equations in two variables.	6.3.1 Solve: $-2x + y = 5$ $2x + 3y = 3$ (a) $(-2, 1)$ (b) $(0, 1)$ (c) $(-\frac{3}{2}, 2)$ (d) $(2, -\frac{3}{2})$
	6.3.2 Solve: $4x - 2y = 9$ $4x + 2y = -1$
6.4 Use multiplication with the addition or subtraction method to solve a system of linear equations.	6.4.1 Solve: $3x + 5y = 2$ $2x - 3y = 1$ (a) $(0, 1)$ (b) $(1, 1)$ (c) $(1, -1)$ (d) $(-1, -1)$
	6.4.2 Solve: $5a + 3b = 12$ $4a - 5b = 17$
6.5 Use systems of linear equations to solve problems.	6.5.1 The sum of the digits of a number is 6. The number is 6 times the units digit. Find the number. (a) 24 (b) 42 (c) 60 (d) 15
	6.5.2 Jason is preparing to run in the Moonlight Marathon. One day he ran and walked a total of 16 miles. If he ran 1 mile more than twice as far as he walked, how many miles did he run?

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 7: The learner will perform operations with polynomials.

OBJECTIVES	SAMPLE MEASURES
7.1 Add and subtract polynomials.	7.1.1 Simplify: $(3x^2 - 5x + 4) - (5x^2 + 7x - 7)$ (a) $8x^2 + 12x + 11$ (b) $-2x^2 - 12x + 11$ (c) $-8x^2 + 2x - 3$ (d) $-2x^2 + 2x - 3$
	7.1.2 Simplify: $(x^2 - 9x + 8) - (3x^2 - 7x + 7)$
	7.1.3 Simplify: $6(3x - y) + 3(x + 2y)$
7.2 Multiply polynomials.	7.2.1 Multiply: $(x - 1)(x^2 + 2x - 3)$ (a) $x^2 + 2x^2 - 3x$ (b) $x^2 - 3x + 5x + 3$ (c) $x^3 + x^2 - x + 3$ (d) $x^2 + x^2 - 5x + 3$
	7.2.2 Multiply: $(x + 2)(3x^2 - x - 1)$
7.3 Divide a polynomial by a monomial.	7.3.1 Divide: $(3x^3 - 6x^2 - 12x) \div (3x)$ (a) $x^2 - 2x - 4$ (b) $x^3 - 2x^2 - 4x$ (c) $x^2 - 2x$ (d) $x^3 + 2x - 4$
	7.3.2 Divide: $(24x^4y^4 - 12x^3y^3 + 28x^2y^3) \div (36xy)$

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 7: The learner will perform operations with polynomials.

OBJECTIVES	SAMPLE MEASURES
7.4 Divide polynomials.	7.4.1 Divide: $(4x^3 - 3x - 1) \div (2x + 1)$ (a) $2x^2 - 1$ (b) $2x^2 + 1$ (c) $2x^2 - x - 1$ (d) $2x^2 + x + 1$ 7.4.2 Divide: $(x^2 + 8x - 20) \div (x - 2)$
7.5 Find the product of two binomials.	7.5.1 Multiply: $(c + 6)(c - 2)$ (a) $c^2 + 4c - 12$ (b) $c^2 - 4c - 12$ (c) $c^2 - 8c - 12$ (d) $c^2 + 4c + 4$ 7.5.2 Multiply: $(2c - 3)(c + 1)$
7.6 Factor a trinomial.	7.6.1 Factor: $x^2 - 5x - 6$ (a) $(x - 6)(x + 1)$ (b) $(x + 6)(x - 1)$ (c) $(x - 2)(x - 3)$ (d) $(x + 2)(x - 3)$ 7.6.2 Factor: $x^2 - 2x - 35$
7.7 Find the square of a binomial.	7.7.1 Multiply: $(m - 5)^2$ (a) $m^2 - 25$ (b) $m^2 - 5m + 25$ (c) $m^2 - 10m - 25$ (d) $m^2 - 10m + 25$ 7.7.2 Multiply: $(m + 6n)^2$

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 7: The learner will perform operations with polynomials.

OBJECTIVES	SAMPLE MEASURES
7.8 Factor a perfect square trinomial.	7.8.1 Factor: $x^2 - 6x + 9$ (a) $(x+3)(x-3)$ (b) $(x+9)(x-1)$ (c) $(x+3)^2$ (d) $(x-3)^2$ 7.8.2 Factor: $4x^2 - 12x + 9$
7.9 Find the product of a sum and a difference.	7.9.1 Multiply: $(y+7)(y-7)$ (a) $y^2 - 14y + 49$ (b) $y^2 - 49$ (c) $y^2 + 14y + 49$ (d) $y^2 + 49$ 7.9.2 Multiply: $(2c+3)(2c-3)$
7.10 Factor the difference of two squares.	7.10.1 Factor: $x^2 - 25$ (a) $(x-5)^2$ (b) $(x-25)(x-1)$ (c) $(x+5)(x-5)$ (d) $(x-10)(x-15)$

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 8: The learner will work with ratios, proportions, and percents.

OBJECTIVES	SAMPLE MEASURES
8.1 Simplify ratios involving algebraic expressions.	8.1.1 Simplify: $\frac{x^2 - 4x}{2x^2 - 5x - 12}$ (a) $\frac{x}{2x - 3}$ (b) $\frac{x - 4}{2x + 3}$ (c) $\frac{x + 4}{2x - 3}$ (d) $\frac{x}{2x + 3}$ 8.1.2 Simplify: $\frac{x^2 - 6x}{x^2 - 36}$
8.2 Solve proportions.	8.2.1 Solve: $\frac{25}{12} = \frac{5x}{18}$ (a) {7.5} (b) {1.5} (c) {15} (d) {0.75} 8.2.2 Solve: $\frac{7x - 5}{4} = \frac{x + 9}{3}$
8.3 Use ratios and proportions to solve problems.	8.3.1 The sales tax on a new car costing \$10,600 is \$424. If the sales tax rate is the same, what is the sales tax on a new car costing \$18,500? (a) \$740 (b) \$750 (c) \$925 (d) \$1,110 8.3.2 The two acute angles in a right triangle have a ratio of 2:3. Find the measure of the larger angle.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 8: The learner will work with ratios, proportions, and percents.

OBJECTIVES	SAMPLE MEASURES
8.4 Solve problems involving percents.	8.4.1 At a pre-season sale, a \$200 suit is on sale for \$150. What is the percent of discount? (a) 15% (b) 20% (c) 25% (d) $33\frac{1}{3}\%$ 8.4.2 A jewelry store purchases a watch for \$80 and marks it up 250%. What is the selling price of the watch?

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 9: The learner will simplify expressions with algebraic fractions.

OBJECTIVES	SAMPLE MEASURES
*9.1 Find the Least Common Multiple (LCM) of algebraic expressions.	9.1.1 The Least Common Multiple for the expressions $x^2 - 9$ and $x^2 - 6x + 9$ is: (a) $(x-3)^2$ (b) $(x-3)^2(x+3)$ (c) $(x+3)^2$ (d) $(x+3)(x-3)$
	9.1.2 Explain why the LCM for $(x-2)(x-3)$ and $(x-2)^2$ is $(x-3)(x-2)(x-2)$.
*9.2 Add algebraic fractions with like denominators.	9.2.1 Simplify: $\frac{5a^2}{a+1} + \frac{a}{a+1} + \frac{5}{a+1}$ (a) $\frac{5a^2+5a}{a+1}$ (b) $\frac{5a^3+5}{a+1}$ (c) $\frac{5a^2+5a}{3a+3}$ (d) $\frac{5a^2+a+5}{a+1}$
	9.2.2 Simplify: $\frac{8t^2}{t-4} + \frac{6t}{t-4} + \frac{7}{t-4}$

* These objectives are not included in the Algebra I curriculum. If time permits, they should be presented to students as a way of showing a connection between arithmetic and algebraic concepts and skills.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 9: The learner will simplify expressions with algebraic fractions.

OBJECTIVES	SAMPLE MEASURES
*9.3 Subtract algebraic fractions with like denominators.	9.3.1 Simplify: $\frac{6k^2}{k^2-1} - \frac{k}{k^2-1}$ (a) $\frac{6k^2-k}{k^2-1}$ (b) $\frac{6k}{k^2-1}$ (c) $\frac{5k}{k^2-1}$ (d) $\frac{6k-1}{k^2-1}$ 9.3.2 Simplify: $\frac{4x+7}{x+1} - \frac{x+4}{x-1}$
*9.4 Add algebraic fractions with unlike denominators.	9.4.1 Simplify: $\frac{a^2+6}{a^2} + \frac{2a}{a+4}$ (a) $\frac{a^2+2a+6}{a^2(a+4)}$ (b) $\frac{2a+6}{a+4}$ (c) $\frac{a^2+8}{a^2}$ (d) $\frac{3a^3+4a^2+6a+24}{a^2(a+4)}$ 9.4.2 Susan had the following answer for this math problem: $\frac{x+3}{x-2} + \frac{x}{x+6} = \frac{2x^2+7x+18}{x^2-12}$ Exlain her error.

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MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 9: The learner will simplify expressions with algebraic fractions.

OBJECTIVES	SAMPLE MEASURES
*9.5 Subtract algebraic fractions with unlike denominators.	9.5.1 Simplify: $\frac{4b}{2x} - \frac{2b}{3x}$ (a) $\frac{4b}{3}$ (b) $\frac{2b}{3}$ (c) $\frac{12bx - 2b}{6x^2}$ (d) $\frac{8bx}{6x^2}$ 9.5.2 Simplify: $\frac{x}{2x+2} - \frac{2}{x+1}$ (a) $\frac{x^2 - 2x + 2}{2(x+1)(x+2)}$ (b) $\frac{x^2 - 2x + 4}{2(x+1)(x+2)}$ (c) $\frac{x - 4}{2(x+1)}$ (d) $\frac{x^2 - 6x + 4}{2(x+1)(x+2)}$

* This objective is not included in the Algebra I curriculum. If time permits, it should be presented to students as a way of showing a connection between arithmetic and algebraic concepts and skills.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 10: The learner will analyze linear equations.

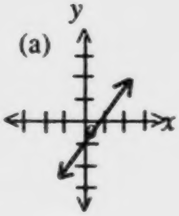
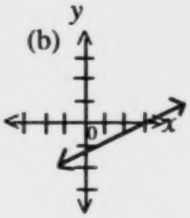
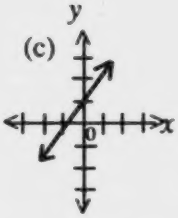
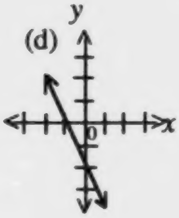
OBJECTIVES	SAMPLE MEASURES
10.1 Write linear equations in slope–intercept form.	10.1.1 Write $2x + y = 3$ in slope–intercept form. (a) $x = -\frac{1}{2}y + 3$ (b) $x = -\frac{1}{2}y + \frac{3}{2}$ (c) $y = 2x - 3$ (d) $y = -2x + 3$
	10.1.2 Write $3x = 2y + 5$ in slope–intercept form.
10.2 Find the slope of a non-vertical line given two points on the line.	10.2.1 Find the slope of a line which passes through (3,5) and (1,4). (a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) -2
	10.2.2 Find the slope of the line which passes through (–1,6) and (2, 7).

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 10: The learner will analyze linear equations.

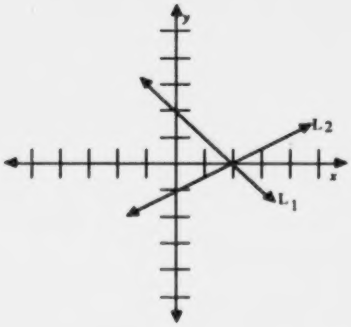
OBJECTIVES	SAMPLE MEASURES
10.3 Graph a line given its slope and y – intercept.	10.3.1 Which is the graph of the line whose slope is $\frac{1}{3}$ and y – intercept is – 1? (a)  (b)  (c)  (d)  10.3.2 Draw the graph of the line whose slope is $-\frac{1}{2}$ and y-intercept is 3.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 10: The learner will analyze linear equations.

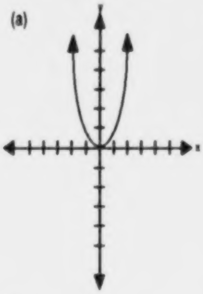
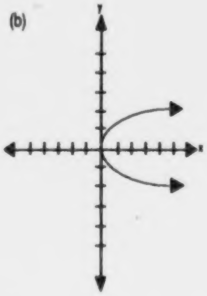
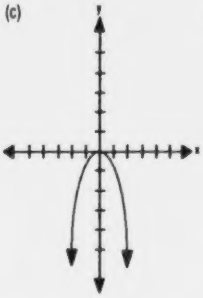
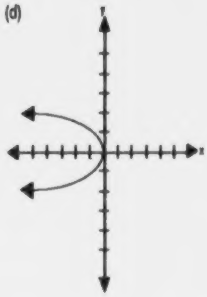
OBJECTIVES	SAMPLE MEASURES
10.4 Determine the slope of a line from its graph.	 10.4.1 L_1 in the graph above has ___ slope. (a) no (b) positive (c) negative (d) 0 10.4.2 What is the slope of L_2 in the graph pictured above?
10.5 Determine the slope of a line from its equation.	10.5.1 The slope of the line whose equation is $2x - y = 4$ is: (a) 2 (b) -2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$ 10.5.2 Find the slope of the line whose equation is $2x - 5y = 10$.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 11: The learner will explore, graph, and interpret non linear equations.

OBJECTIVES	SAMPLE MEASURES
11.1 Graph a quadratic equation.	11.1.1 Which is the graph of $y = x^2$? (a)  (b)  (c)  (d) 
	11.1.2 Draw the graph of $y = x^2 - x - 2$ by a method of your choice.
11.2 Use an automatic grapher to find the solution to a quadratic equation.	11.2.1 Use an automatic grapher to solve $4x^2 = 5 - 8x$. (a) $\{-2.5, 0.5\}$ (b) $\{0.5, 2.5\}$ (c) $\{-0.5, 2.5\}$ (d) $\{-2.5, -0.5\}$
	11.2.2 Use an automatic grapher to solve $1.1x^2 + 3.2x = 1.2$.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 11: The learner will explore, graph, and interpret non linear equations.

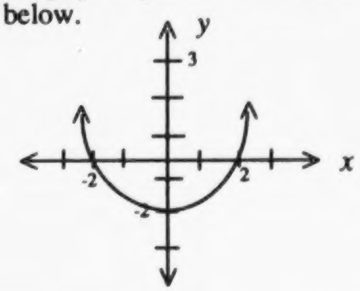
OBJECTIVES		SAMPLE MEASURES
11.3	Solve a quadratic equation when one member is in factored form and the other member is zero.	11.3.1 Solve: $(x - 2)(2x - 3) = 0$ (a) $\{2, 3\}$ (b) $\{2, -3\}$ (c) $\{-2, 3\}$ (d) $\{\frac{3}{2}, 2\}$ 11.3.2 Explain how to solve: $(3x + 5)(x - 6) = 0$.
11.4	Solve a quadratic equation in which a perfect square equals a constant.	11.4.1 Solve: $x^2 = 36$ (a) $\{6, 6\}$ (b) $\{-6, 6\}$ (c) $\{1, 36\}$ (d) $\{1, -36\}$ 11.4.2 Solve: $(x - 3)^2 = 36$
11.5	Solve a quadratic equation by factoring.	11.5.1 Solve: $x^2 - 2x - 35 = 0$ (a) $\{5, -7\}$ (b) $\{7, -5\}$ (c) $\{5, 7\}$ (d) $\{7, -5\}$ 11.5.2 The equation $x^2 + 3x = 54$ is true when $x = 6$. What value of x will make $(x - 2)^2 + 3(x - 2) = 54$ true?

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 11: The learner will explore, graph, and interpret non linear equations.

OBJECTIVES	SAMPLE MEASURES
	11.5.3 Use an automatic grapher to find the *x*-intercepts of $y = x^2 - x - 12$. Solve the equation $x^2 - x - 12 = 0$ by factoring. How are the answers to these two exercises related? Why is this reasonable?
11.6 Understand that the vertex provides the maximum or minimum value of the function.	11.6.1 The graph of $y = x^2 - 2$ is shown below.  What is the smallest *y*-value on the graph? (a) –2 (b) 2 (c) 0 (d) –6
	11.6.2 The height *h* of a toy rocket *t* seconds after launching is determined by the formula $h = 16t^2 + 112t$. Use an automatic grapher to find the greatest height attained by the rocket. Then find when the rocket hits the ground again.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 11: The learner will explore, graph, and interpret non linear equations.

OBJECTIVES	SAMPLE MEASURES
11.7 Solve a quadratic equation by using the quadratic formula.	11.7.1 Use the quadratic formula to solve $x^2 - 4x + 1 = 0$. (a) $\{-2 + \sqrt{3}, -2 - \sqrt{3}\}$ (b) $\{2 + \sqrt{3}, 2 - \sqrt{3}\}$ (c) $\{-4, 1\}$ (d) $\{-5, 1\}$
	11.7.2 Use the quadratic formula to solve $2x^2 - 5x = -2$.
11.8 Use quadratic equations to solve problems.	11.8.1 The sum of a number and its square is 72. Find the numbers. (a) –9, 8 (b) –8, 9 (c) 9 (d) 8
	11.8.2 Use an automatic grapher to solve $1.4x^2 - 0.7x = 0.2$.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 11: The learner will explore, graph, and interpret non linear equations.

OBJECTIVES	SAMPLE MEASURES
11.9 Determine if a set of data represents an exponential function.	11.9.1 Which of the following tables of data exhibit exponential behavior?

x	*y*
0	1
2	5
4	9
6	13
8	17
10	21

x	*y*
1	1
2	2.1
3	3.3
4	4.6
5	5.9
6	7.2

x	*y*
1	600
2	300
3	200
4	150
5	120
6	100

x	*y*
0	2
2	4
4	8
6	12
8	16
10	20

11.9.2 Explain to a friend how to test a set of data to determine if it represents exponential behavior.

MATHEMATICS

Algebra I - B

GRADE LEVEL: 8-12

COMPETENCY GOAL 11: The learner will explore, graph, and interpret non linear equations.

OBJECTIVES	SAMPLE MEASURES
11.10 Use formulas, calculators, and automatic graphers to explore and solve problems involving exponentials.	11.10.1 Evaluate $p(1 + r)^t$ to the nearest whole number when $p = 120$, $r = 08$, and $t = 20$. (a) \$56 (b) \$559 (c) \$176 d) \$125 11.10.2 Ann Elmore deposits \$8500 in the bank, where the interest rate is 8% per year. How much money will she have at the end of 10 years? (Use the formula $A = p(1 + r)^t$, where A = the amount, p = the principal, t = the time, and r = the interest rate.)

NOTES

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 1: The learner will use the language of algebra.

OBJECTIVES	SAMPLE MEASURES
1.1 Evaluate algebraic expressions.	1.1.1 Evaluate $ab - cd$ if $a = 2, b = 3, c = 4$, and $d = 0$. (a) 10 (b) 2 (c) 5 (d) 6
	1.1.2 Use two or more of the variables above and their assigned values to write an expression whose value is 10.
	1.1.3 If $x = 5$, which is larger, $x^2 + x$ or -25?
1.2 Use formulas to solve problems.	1.2.1 Use the formula $D = rt$ to find the distance (D), when $r = 55$ mi/h and $t = 2h$. (a) 57 mi (b) 100 mi (c) 110 mi (d) 27.5 mi
	1.2.2 Mark used the formula above to determine how long it should take him to drive 275 mi at 55 mi/h. He got 220 h for his answer. Explain his error.
	1.2.3 The formula relating Farenheit and Celsius is: $F = \frac{9}{5}C + 32.$ Tell a friend how you would find the temperature in Farenheit if you see a thermometer indicating it is 20° Celsius.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 1: The learner will use the language of algebra.

OBJECTIVES		SAMPLE MEASURES	
1.3	Translate word phrases and sentences into expressions and equations and vice versa.	1.3.1	Translate into a variable expression: 5 more than twice a number *n*. (a) $2n + 5$ (b) $2(n + 5)$ (c) $2 + 5n$ (d) $5(n + 2)$
		1.3.2	Write as an equation: *y* is three more than *x*.
		1.3.3	Write the following equation as a sentence: $y = 2x$.
1.4	Use the associative, commutative and distributive properties.	1.4.1	Give an example to show that subtraction is not commutative.
		1.4.2	Use mental computation and any of the number properties you know to simplify: $3.75x + 1.95x + 2.05x + 10.25x$.
		1.4.3	Simplify: $2(3x - 5)$ (a) $5x - 7$ (b) $6x - 5$ (c) $6x - 10$ (d) $5x - 5$
		1.4.4	$a * b$ is defined for all real numbers *a* and *b* by $a * b = 2a + b$. Is * commutative?

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 2: The learner will perform operations with real numbers.

OBJECTIVES	SAMPLE MEASURES
2.1 Simplify real number expressions with and without a calculator.	2.1.1 Simplify: $\frac{284 + 8^2}{16}$ (a) 21.75 (b) 5329 (c) 288 (d) 81.75
	2.1.2 Explain the sequence of keystrokes you would use to find this value on your calculator: $\frac{2}{3}(2^5 - 3^2)$
2.2 Determine the additive or multiplicative inverse of a number.	2.2.1 Which is the multiple inverse of (-3)? (a) -3 (b) $\frac{1}{3}$ (c) $-\frac{1}{3}$ (d) 3
	2.2.2 For what value of n is $n = -n$?
2.3 Determine the absolute value of expressions.	2.3.1 Simplify: $\lvert 2 - 7 \rvert$ (a) -5 (b) 5 (c) 9 (d) - 9
	2.3.2 For what value of n is $\lvert n \rvert = -\lvert n \rvert$?
2.4 Raise a real number to an indicated power.	2.4.1 Simplify: $(-2)^5$ (a) -10 (b) -32 (c) 10 (d) 32
	2.4.2 Give examples to disprove this statement: "The square of a number is always larger than the number."

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 2: The learner will perform operations with real numbers.

OBJECTIVES	SAMPLE MEASURES
2.5 Write numbers in scientific notation and use this notation with a calculator.	2.5.1 The length of time it takes a super computer to do a calculation is 1×10^{-9} seconds. Rewrite this time in decimal form.
	2.5.2 What sequence of keystrokes would you use on your calculator to enter the number 1.4×10^4?
	2.5.3 Use you calculator to complete the product of 4,520,000 and 853,000,000 and express your answer in scientific notation.
2.6 Distinguish between rational and irrational numbers.	2.6.1 Which is an irrational number? (a) $4.\overline{2}$ (b) $\sqrt{2}$ (c) $\sqrt{4}$ (d) -4.2
	2.6.2 Give an example of an irrational number between 2 and 10.
2.7 Find approximations for square roots with and without a calculator.	2.7.1 Between what two integers does $\sqrt{88}$ lie?
	2.7.2 If 289 is a perfect square, then what kind of a number is $\sqrt{289}$?
	2.7.3 Use your calculator to approximate $\sqrt{391}$ to the nearest hundredth.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 2: The learner will perform operations with real numbers.

OBJECTIVES	SAMPLE MEASURES
2.8 Simplify radical expressions.	2.8.1 Simplify: $2\sqrt{112}$ (a) $8\sqrt{7}$ (b) $7\sqrt{8}$ (c) $4\sqrt{28}$ (d) $8\sqrt{17}$
	2.8.2 Simplify: $\sqrt{16} \cdot \sqrt{25}$ (a) 20 (b) 100 (c) 80 (d) 9
	2.8.3 Simplify: $\dfrac{\sqrt{320}}{\sqrt{20}}$
	2.8.4 Simplify: $5\sqrt{3} + 2\sqrt{75} - \sqrt{27}$
2.9 Multiply two binomials which contain square roots.	2.9.1 Multiply: $(2 + \sqrt{3})(2 - \sqrt{3})$ (a) -1 (b) $4\sqrt{3}$ (c) $4 - \sqrt{3}$ (d) 1
	2.9.2 Multiply: $(4 - \sqrt{5})^2$
2.10 Compare real number expressions.	2.10.1 Replace the ? with <, =, or > to make a true statement: $\sqrt{64} + \sqrt{25} + \sqrt{16} \ \underline{?} \ \sqrt{64 + 25 + 16}$ (a) = (b) < (c) > (d) none of these
	2.10.2 Give a value for n so that $\frac{1}{n} > n$.

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MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 3: The learner will solve equations and inequalities with one variable.

OBJECTIVES		SAMPLE MEASURES	
3.1	Solve an equation by using the addition property of equality and the idea of additive inverse.	3.1.1	Solve: $x + 9 = 15$ (a) {6} (b) {24} (c) {−6} (d) {−24}
		3.1.2	If $x - 7 = 12$, then $x + 2 = \underline{?}$.
3.2	Solve a simple equation by using the multiplication property of equality and the idea of multiplicative inverse.	3.2.1	Solve: $12x = 72$ (a) {6} (b) {-6} (c) {84} (d) {-60}
		3.2.2	Write an equation which could be solved by the multiplication property of equality.
3.3	Solve an equation graphically and by using more than one property of equality.	3.3.1	Solve: $2t + 1 = 19$ (a) {−9} (b) {10} (c) {−10} (d) {9}
		3.3.2	Explain the procedure you would use to solve $4x - 3 = 25$.
		3.3.3	Use an automatic grapher to solve $13x - 43 = 19$ to the nearest tenth.
3.4	Solve an equation which contains similar terms.	3.4.1	Solve: $6x - 2x - 4 = 24$ (a){5} (b) {−5} (c) {7} (d) {−7}
		3.4.2	Explain why 3 is not the solution to the equation $5x + 3x - 7 = 31$.
3.5	Solve an equation which has the variable in both members.	3.5.1	Solve: $n - 5 = 19 - 3n$ (a){-12} (b){12} (c) {6 (d) {-6}
		3.5.2	Explain the procedure you would use to solve $5x + 3 = 2x - 9$.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 3: The learner will solve equations and inequalities with one variable.

OBJECTIVES	SAMPLE MEASURES
3.6 Solve an equation in which the numerical coefficient is a fraction.	3.6.1 Solve: $\frac{2}{3}x - 5 = \frac{x}{4} - 10$ (a) {-12} (b) {12} (c) {-1} (d) {-2}
	3.6.2 Solve: $\frac{2}{3}x - 9 = x$
3.7 Solve a formula for one of its variables or find the value of a variable when values of the other variables are given.	3.7.1 Solve $by + c = a$ for b. (a) $b = \frac{a + c}{y}$ (b) $b = \frac{a - y}{c}$ (c) $b = \frac{a - c}{y}$ (d) $b = \frac{a + y}{c}$
	3.7.2 The circumference of a circle can be found by the formula $C = 2\pi r$. Rewrite this formula to solve for the radius.

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MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 3: The learner will solve equations and inequalities with one variable.

OBJECTIVES	SAMPLE MEASURES
3.8 Use problem solving skills to solve real world and "word" problems which involve a linear equation or a formula.	3.8.1 The lengths of the legs of a right triangle are 9m and 12m. Use the Pythagorean Theorem to find the length of the hypotenuse. (a) 21m (b) 15m (c) 16m (d) 18m
	3.8.2 Your grandmother tells you that if the temperature is above 45^0 the number of times a cricket chirps in a minute is related to the temperature. In particular, the number of chirps is obtained by multiplying the temperature by 4 and adding 20. Find the temperature if the number of chirps per minute is 320. (a) 85^0 (b) 60^0 (c) 75^0 (d) 100^0
	3.8.3 You are paid \$2.25 per hour for babysitting and an extra \$4.00 for putting the children to bed. Write a formula to determine the amount you would earn in h hours assuming you put the children to bed.
3.9 Solve a simple equation involving absolute value.	3.9.1 Solve: $\lvert x - 5 \rvert = 4$ (a) both 1 and 9 (b) 9 only (c) 1 only (d) No solution
	3.9.2 Explain why $\lvert x + 4 \rvert = -7$ has no solution.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 3: The learner will solve equations and inequalities with one variable.

OBJECTIVES	SAMPLE MEASURES
3.10 Solve a simple equation containing a radical.	3.10.1 Solve: $\sqrt{5x + 1} = 4$ (a) $\{1\}$ (b) $\{2\}$ (c) $\{3\}$ (d) no solution
	3.10.2 Explain why it is important to check your answer when you solve an equation containing a radical.
3.11 Find the solution set for a linear inequality when replacement values are given for the variable.	3.11.1 Solve $2x > -4$ if $x \in \{-3, -2, -1, 0, 1, 2, 3\}$. (a) $\{-1, 0, 1, 2, 3\}$ (b) $\{3\}$ (c) $\{-3\}$ (d) $\{-2, -1, 0, 1, 2, 3\}$
	3.11.2 Solve $3x + 1 < 4$ if $x \in \{-3, -2, -1, 0, 1, 2, 3\}$.
3.12 Solve a linear inequality by using transformations.	3.12.1 Solve: $4x - 1 \geq 2x + 5$ (a) $x > 3$ (b) $x \geq -3$ (c) $x \leq -3$ (d) $x \geq 3$
	3.12.2 Sandra got $x \leq -5$ for the answer to $3x + 2 \leq 4x + 7$. Identify her error.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 3: The learner will solve equations and inequalities with one variable.

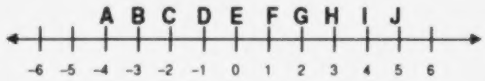
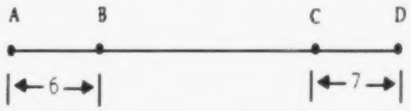
OBJECTIVES	SAMPLE MEASURES
3.13 Use inequalities to solve problems.	3.13.1 Some children have put up a lemonade stand. They gave their mother \$2.00 for the lemonade. If they sell a glass of lemonade for \$.15 per glass, how many glasses must they sell if they want to make a profit? How many glasses must they sell if they want to make a profit of more than \$1.00?
	3.13.2 Fran has scored 16, 13, and 30 points in her last three games. How many points must she score in the next game so that her four game average does not fall below 20 points? (a) 19 (b) 21 (c) 22 (d) 20
3.14 Find the solution set of combined inequalities.	3.14.1 Solve: $-3 \le x - 1 < 4$ (a) $-4 \le x < 5$ (b) $-4 \le x < 3$ (c) $-2 \le x < 5$ (d) $-2 \le x < 3$
	3.14.2 Solve: $-5 < 1 - 3x \le 7$

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 4: The learner will demonstrate an elementary understanding of relations and functions.

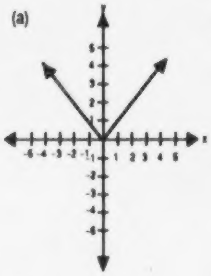
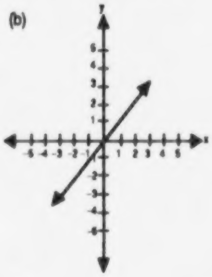
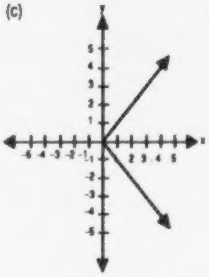
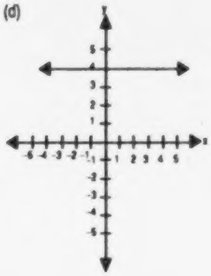
OBJECTIVES		SAMPLE MEASURES
4.1	Graph and locate sets of real numbers on a number line.	List the letter for the point whose coordinate is given:  4.1.1 –3 (a) H (b) B (c) E (d) C 4.1.2 Locate $-\frac{2}{3}$ on the number line above and label it point *K*.
4.2	Graph ordered pairs of numbers on the coordinate plane and interpret information related to these sets of points.	4.2.1 In which quadrant is the graph of (-2, -6)? (a) I (b) II (c) III (d) IV 4.2.2 Plot the points (0, 2), (3, 0), and (6, -2) in a coordinate plane to determine if they are collinear.
4.3	Find the distance between two points on a number line.	4.3.1 On a number line, if point *M* has coordinate -4 and point *N* has coordinate 3, how long is *MN*? 4.3.2  In the figure above, if segment AC has length 14, then find the length of segment BD.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 4: The learner will demonstrate an elementary understanding of relations and functions.

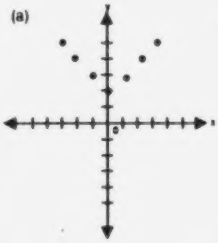
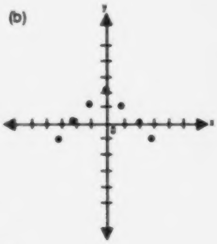
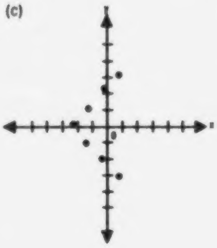
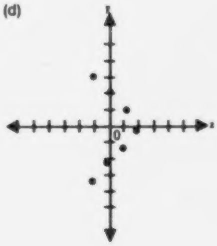
OBJECTIVES		SAMPLE MEASURES	
4.4	Graph a relation on the coordinate plane.	4.4.1	What is the graph of the relation $A=\{(5, 1), (-3, 2), (3, -2)\}$? (a) a point (b) three points on the coordinate plane (c) a line (d) three line segments
		4.4.2	Graph the relation: $B = \{(-3, 2), (0, 3), (3, 3), (3, -2)\}$
4.5	Distinguish between a relation and a function.	4.5.1	Tell which graph is not a function. (a)  (b)  (c)  (d) 
		4.5.2	Give an example to illustrate the truth of the statement that "every function is a relation but every relation is not a function."

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 4: The learner will demonstrate an elementary understanding of relations and functions.

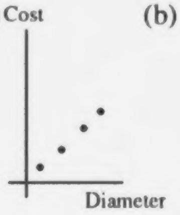
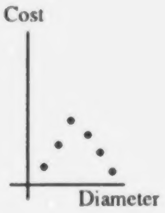
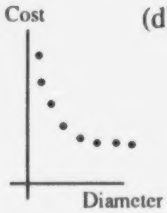
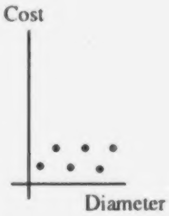
OBJECTIVES	SAMPLE MEASURES
4.6 Graph a relation given an equation and domain.	4.6.1 Which is the graph of $y = \lvert x \rvert + 2$ when $x \in \{-3, -2, -1, 0, 1, 2, 3\}$? (a)  (b)  (c)  (d) 
	4.6.2 Draw the graph of $y = \lvert x \rvert + 2$ when $x \in \{\text{real numbers}\}$. Explain how this differs from the graph in 4.6.1.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 4: The learner will demonstrate an elementary understanding of relations and functions.

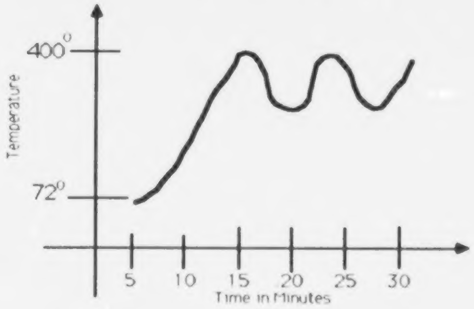
OBJECTIVES	SAMPLE MEASURES
4.7 Sketch a reasonable graph for a given relationship.	4.7.1 Which graph best represents the relationship between the cost of pizzas of various diameters? (a)  (b)  (c)  (d) 
	4.7.2 Sketch a reasonable graph which represents an individual's height as he ages. Label both axes and write a sentence or two to justify the shape or behavior of your graph.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 4: The learner will demonstrate an elementary understanding of relations and functions.

OBJECTIVES	SAMPLE MEASURES
4.8 Interpret a graph in a real-world setting.	4.8.1 The graph of the temperature of an oven over a 30-minute period is provided below. At about what time is the temperature 200° F?  (a) 5 minutes (b) 10 minutes (c) 15 minutes (d) 20 minutes
	4.8.2 The graph of the temperature of an oven over a 30-minute period of time is provided above. (a) Why is the early temperature 72°? (b) What is represented over the time period from 5 to 15 minutes? (c) What is represented over the time period from 15 to 30 minutes? (d) If the oven is turned off after 40 minutes, sketch a graph of temperature vs. time over the period of time from 40 to 100 minutes.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 4: The learner will demonstrate an elementary understanding of relations and functions.

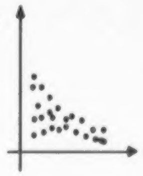
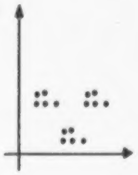
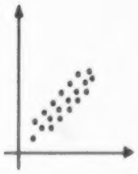
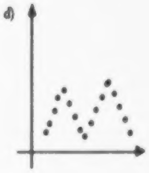
OBJECTIVES	SAMPLE MEASURES
4.9 Use a computer or graphing calculator to explore the graphs of functions.	4.9.1 Graph $y = x$ and $y = 3x + 5$ using an automatic grapher. How many points of intersection exist? (a) 0 (b) 1 (c) 2 (d) 3
	4.9.2 Graph $y = \lvert x \rvert$ and $y = \lvert x \rvert + 3$ using an automatic grapher. Describe how the two graphs differ.
4.10 Compare ordered pairs to the line $y = x$ and interpret the results.	4.10.1 Five students begin training for soccer. Prior to the training program, the students' weights were 128, 132, 154, 165 and 170 pounds. After the training program, the students' weights were 134, 140, 152, 165, and 165 respectively. How many of the ordered pairs in this data set fall on, above, or below the line $y = x$? (a) All above (b) 2 above, 2 below, 1 on (c) All on (d) 4 above, 1 on
	4.10.2 Describe how you know when an ordered pair such as (5, 12) is above, on, or below the line $y = x$.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 5: The learner will graph and use linear equations and inequalities.

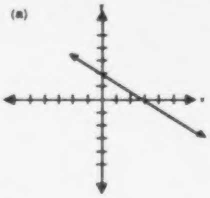
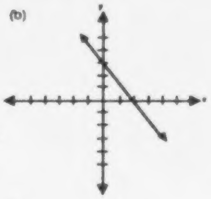
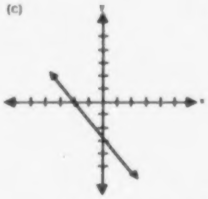
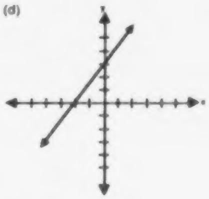
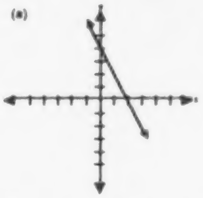
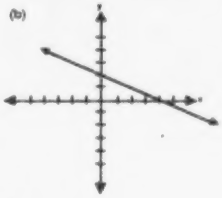
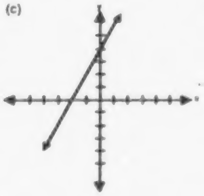
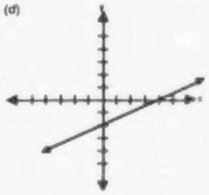
OBJECTIVES	SAMPLE MEASURES
5.1 Determine if data are behaving in a linear fashion.	5.1.1 Which data appear linear? a)  b)  c)  d)  5.1.2 Describe a relationship between two variables that you believe would behave in a linear fashion.
5.2 Find the solution set of open sentences in two variables when given replacement sets for the variables.	5.2.1 Which ordered pair lies on the line $2x - 5y = -4$? (a) (-3, 2) (b) (3, -2) (c) $(\frac{1}{2}, 1)$ (d) (0, -2) 5.2.2 Show why (-2, -1) is or is not a solution of $x - 3y = 1$.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 5: The learner will graph and use linear equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
5.3 Graph a linear equation in two variables.	5.3.1 Which is the graph of $2x + 3y = 6$?  (a)  (b)  (c)  (d)
	5.3.2 Draw the graph of $x + 2y = 2$.
5.4 Graph a line given its slope and *y*-intercept	5.4.1 Which is the graph of the line whose slope is -2 and *y* –intercept is 4?  (a)  (b)  (c)  (d)
	5.4.2 Draw the graph of the line whose slope is $-\frac{1}{2}$ and *y* –intercept is 1.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 5: The learner will graph and use linear equations and inequalities.

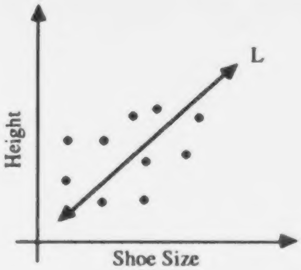
OBJECTIVES	SAMPLE MEASURES
5.5 Find the slope of a non-vertical line given the graph of a line or an equation of the line or two points on the line.	5.5.1 Find the slope of the line that passes through the points (-2, 4) and (3, -1). (a) -1 (b) 1 (c) $\frac{1}{3}$ (d) -3 5.5.2 Explain how you can determine that the graph of one line is steeper than the graph of another without actually drawing the graphs.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 5: The learner will graph and use linear equations and inequalities.

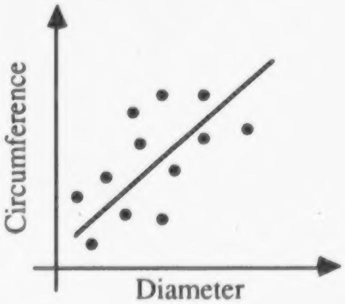
OBJECTIVES	SAMPLE MEASURES
5.6 Describe the slope in a real-world linear relationship using real–world terms.	 5.6.1 Interpret the slope of L if the slope value is 2/3. (a) For every increase of 2" in height, there is a corresponding increase of 3 shoe sizes. (b) For every increase of 2 shoe sizes, there will be a corresponding decrease of 3 shoe sizes. (c) For every increase of 2" in height, there will be a corresponding decrease of 3 shoe sizes. (d) For every decrease of 2" in height, there will be a corresponding increase of 3 shoe sizes.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 5: The learner will graph and use linear equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
	5.6.2 After gathering a lot of data about the circumference and diameter of tin cans, the following graph was made. The slope of the data was estimated to be 2.98. Write a sentence using the words "circumference", "diameter" and the number 2.98 which describes how the three are related. 
5.7 Write the slope-intercept form of an equation of a line.	5.7.1 Write $2x - y = 4$ in slope-intercept form. (a) $y = 2x + 4$ (b) $x = \frac{1}{2}y + 4$ (c) $y = 2x - 4$ (d) $x = \frac{1}{2}y + 2$ 5.7.2 Find the slope and y- intercept of $2x - 4y = 1$.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 5: The learner will graph and use linear equations and inequalities.

OBJECTIVES	SAMPLE MEASURES

5.8 Write the equation of a line given the slope and one point on the line, or two points on the line.

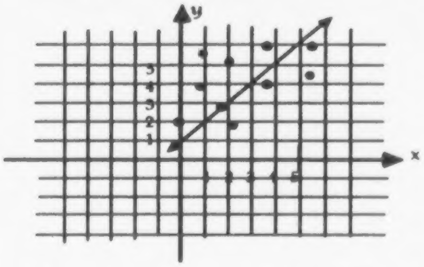
5.8.1 Write an equation of the line which passes through the point (2, 3) and has a slope of -1.

(a) $x + y = 5$ (b) $x - y = -1$
(c) $x - y = 5$ (d) $x + y = 1$

5.8.2 Write an equation of the line which passes through the points (-1, 11) and (1, 5).

5.9 Write the equation of a line which models a set of real data.

5.9.1 John drew the line below to represent his data. What equation would the line have?



(a) $y = x + 1$ (b) $y = -x + 2$
(c) $y = 2x - 1$ (d) $y = x$

5.9.2 This set of data was found by some students who measured their necks and wrists in centimeters to determine if there is a correlation.

wrist size	15.6	15.6	15.8	16.3	16.9	17.3
neck size	34.2	36.0	35.6	38.1	40.1	36.8

Graph the data set and sketch a line that fits the data. Write the equation of the line.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 5: The learner will graph and use linear equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
5.10 Use the line which models real data to make predictions.	5.10.1 The equation of the line for the set of ordered pairs (height, armspan) is $a = 0.94h + 1.3$, measured in inches. What does this equation predict is the approximate armspan of a person who is 62 inches tall? (a) 55 inches (b) 60 inches (c) 65 inches (d) 70 inches
	5.10.2 Given the following data collected about the age of cars in the school parking lot and the number of miles on their odometers, predict the number of miles on the odometer of a 10- year old car. Justify your prediction.

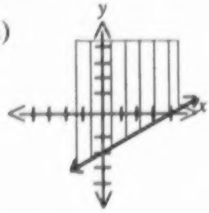
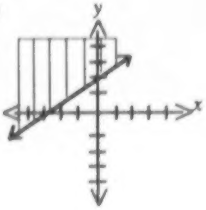
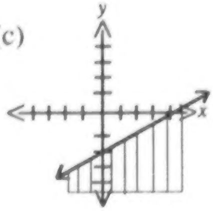
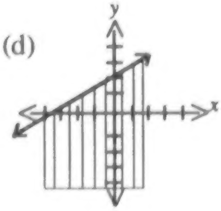
Age (years)	Miles
1	13,065
4	47,142
5	62,066
7	82,863
8	96,410

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 5: The learner will graph and use linear equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
5.11 Graph a linear inequality in two variables.	5.11.1 Which is the graph of the inequality $x - 2y \le 4$? (a)  (b)  (c)  (d) 
	5.11.2 Draw the graph of $y > 2x + 4$.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 6: The learner will graph and solve systems of linear equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
6.1 Use a graph to find the solution of a pair of linear equations in two variables.	6.1.1 Solve by using a graph: $x + y = 6$ $2x - y = 6$ (a) (4, 2) (b) (2, 4) (c) (3, 3) (d) (3, 0)
	6.1.2 Graph the following system of equations: $x - y = 5$ $x = 3 - y$ Name the point of intersection.
6.2 Graph the solution set of a system of linear inequalities in two variables.	6.2.1 Which point does not belong to the graph of the solution set of the system below? $y \geq 2$ $x - y \geq 2$ (a) (0, 0) (b) (4, 2) (c) (8, 2) (d) (-4, -2)
	6.2.2 Graph the solution set of the system: $y \geq 3$ $x - y \geq 3$

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 6: The learner will graph and solve systems of linear equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
6.3 Use a computer or graphics calculator to solve systems of linear equations.	6.3.1 Use a computer or graphics calculator to find the solution of this system to the nearest tenth: $2x - 3y = 1$ $7x + 4y = 12$ (a) (1.8, 3.6) (b) (-1.4, 4.6) (c) (-6, 2.4) (d) (1.4, .6)
	6.3.2 Use a computer or graphics calculator to find the approximate solution of this system: $7x - y = 1$ $x + 4y = 8$
6.4 Use the substitution method to find the solution of a pair of linear equations in two variables.	6.4.1 Solve by substitution: $3x - y = 17$ $2x + y = 8$ (a) (3, 2) (b) (4, 5) (c) (6, 1) (d) (5, -2)
	6.4.2 Demonstrate how to use the substitution method in solving this system. $x + 4y = 9$ $3x - y = 1$

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 6: The learner will graph and solve systems of linear equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
6.5 Use the addition or subtraction method to find the solution of a pair of linear equations in two variables.	6.5.1 Solve by the addition or subtraction method: $x + 2y = 5$ $3x - 2y = -1$ (a) (1, 2) (b) (-1, 3) (c) (3, 1) (d) $(2, -\frac{1}{2})$ 6.5.2 If $x + y = 12$ and $x - y = 6$, then $xy =$ ______.
6.6 Use multiplication with the addition or subtraction method to solve systems of linear equations.	6.6.1 Solve by using multiplication with the addition or subtraction method: $2x + 3y = 1$ $5x - 6y = 16$ (a) (-1, 1) (b) $(3, -\frac{1}{6})$ (c) (2, -1) (d) (-1, 2) 6.6.2 Given the system of equations below: $2x + 5y = 1$ $3x + 2y = -7$ Explain the procedure involved to eliminate the variable *y* when using multiplication with the addition or subtraction method of solving systems of linear equations.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 6: The learner will graph and solve systems of linear equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
6.7 Use systems of linear equations to solve problems.	6.7.1 Mr. Green brought dinner home to his family from the local fast food restaurant. He brought hotdogs, which sell for $1.50 each, and hamburgers, which sell for $2.00 each. If he bought a total of 8 hamburgers and hotdogs, and he spent $14.50, how many hamburgers and how many hotdogs did he purchase? (a) 5 hamburgers and 3 hotdogs (b) 2 hamburgers and 7 hotdogs (c) 4 hamburgers and 4 hotdogs (d) all hotdogs
	6.7.2 The sum of the digits of a two-digit number is 9. If the digits are reversed the new number is 27 more that the original number. Find the original number.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 7: The learner will perform operations with polynomials.

OBJECTIVES	SAMPLE MEASURES
7.1 Add and subtract polynomials.	7.1.1 Simplify: $(6x^2 - 4x - 3) + (x^2 + x - 5)$ (a) $7x^2 - 3x - 8$ (c) $5x^2 - 5x + 2$ (b) $7x^2 - 3x + 8$ (d) $6x^4 - x^2 - 8$
	7.1.2 Give an example of two like terms which contain powers of x and y. Then show the result of subtracting one of them from the other.
7.2 Multiply monomials.	7.2.1 Simplify: $(5x^2y)(-4xy^2)(-x^3y^3)$ (a) $-20x^6y^6$ (b) $20x^5y^5$ (c) $-20x^5y^5$ (d) $20x^6y^6$
	7.2.2 Simplify: $(3n^5)(6n^3)(2n)$
7.3 Find an indicated power of a monomial.	7.3.1 Simplify: $(-4a^3)^2$ (a) $16a^5$ (b) $-16a^5$ (c) $16a^6$ (d) $16a^9$
	7.3.2 Simplify: $(-x^2y)^3$

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 7: The learner will perform operations with polynomials.

OBJECTIVES	SAMPLE MEASURES
7.4 Multiply a polynomial by a monomial.	7.4.1 Multiply: $3a^4(a^3 - 2a^2 + 7)$ (a) $3a^7 - 42a^6$ (b) $3a^7 - 6a^6 + 21a^4$ (c) $3a^7 - 27a^6$ (d) $3a^{12} - 6a^8 + 21a^4$ 7.4.2 Simplify: $x^2(x^2 - 2x) - 3x^2$
7.5 Find the product of two binomials.	7.5.1 Multiply: $(c - 8)(c + 2)$ (a) $c^2 - 10c - 16$ (b) $c^2 - 6c - 10$ (c) $c^2 - 6c - 16$ (d) $c^2 + 6c - 16$ 7.5.2 Multiply: $(c + 3)^2$
7.6 Multiply two polynomials.	7.6.1 Multiply: $(x + 1)(x^2 - x - 2)$ (a) $x^3 - 2x^2 - 3x - 2$ (b) $x^3 - 3x - 2$ (c) $x^3 - x^2 - 2x - 2$ (d) $x^3 + 2x^2 - x - 2$ 7.6.2 Multiply: $(a - 3)(a^2 - 6a - 7)$

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 7: The learner will perform operations with polynomials.

OBJECTIVES	SAMPLE MEASURES
7.7 Divide two monomials.	7.7.1 Divide: $\frac{96x^6}{12x^2}$ (a) $8x^4$ (b) $8x^3$ (c) $\frac{x^4}{8}$ (d) $\frac{8}{x^4}$
	7.7.2 Divide: $\frac{75x^3y^2}{-5xy^2}$
7.8 Divide a polynomial by a monomial.	7.8.1 Divide: $\frac{10a^2b^3 - 15a^3b^2}{5a^2b^2}$ (a) $2b - 3a^3b^2$ (b) $2a^2b - 3ab$ (c) $2ab - 3$ (d) $2b - 3a$
	7.8.2 Divide: $\frac{3x^4y^3 - 12x^3y^5 + 24x^2y^7}{3x^2y^3}$
7.9 Find a common monomial factor in a polynomial.	7.9.1 Find the greatest monomial factor in $8x^2y - 24xy - 40x^2y^2$ (a) $8x$ (b) $8xy$ (c) 8 (d) $8xy^2$
	7.9.2 Factor: $42a^2b^2 - 70a^3b + 28a^2$

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 7: The learner will perform operations with polynomials.

OBJECTIVES	SAMPLE MEASURES
7.10 Factor the difference of two squares.	7.10.1 Factor: $x^2 - 100$ (a) $(x + 10)(x - 10)$ (b) $(x - 10)^2$ (c) $(x + 5)(x - 20)$ (d) $(x - 25)(x + 4)$
	7.10.2 Factor: $4x^2 - 25$
7.11 Factor a simple quadratic trinomial.	7.11.1 Factor: $x^2 - 4x - 12$ (a) $(x - 6)(x + 2)$ (b) $(x - 6)(x - 2)$ (c) $(x + 6)(x - 2)$ (d) $(x - 4)(x + 3)$
	7.11.2 Factor: $4x^2 - 20x + 25$
	7.11.3 Factor: $3x^2 - 2x - 1$
	7.11.4 Factor: $2x^2 - 2x - 12$

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 8: The learner will work with ratios, proportions, and percents.

OBJECTIVES	SAMPLE MEASURES
8.1 Simplify ratios involving algebraic expressions.	8.1.1 Simplify: $\frac{x^2 - 4x}{2x^2 - 5x - 12}$ (a) $\frac{x}{2x - 3}$ (b) $\frac{x - 4}{2x + 3}$ (c) $\frac{x + 4}{2x - 3}$ (d) $\frac{x}{2x + 3}$ 8.1.2 Simplify: $\frac{x^2 + 6x}{x^2 - 36}$
8.2 Solve proportions.	8.2.1 Solve: $\frac{25}{12} = \frac{5x}{18}$ (a) {7.5} (b) {1.5} (c) {15} (d) {0.75} 8.2.2 Solve: $\frac{7x - 5}{4} = \frac{x + 9}{3}$

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 8: The learner will work with ratios, proportions, and percents.

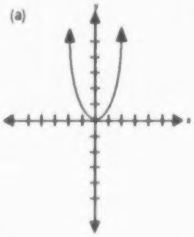
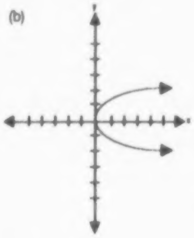
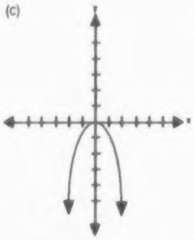
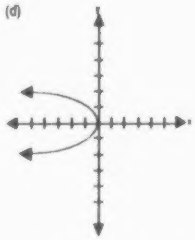
OBJECTIVES	SAMPLE MEASURES
8.3 Use ratios and proportions to solve problems.	8.3.1 Frank and Bradley work together as landscapers. If their earnings are in the ratio 3 to 4 respectively, how much does Bradley earn when Frank earns $720? (a) $540 (b) $850 (c) $680 (d) $960
	8.3.2 The sales tax on a new car costing $10,600 is $424. If the sales tax rate is the same, what is the sales tax on a new car costing $18,500?
	8.3.3 A tree casts a 16-meter shadow at the same time of day that a 3-meter vertical stick casts a 2-meter shadow. How tall is the tree? (a) 21 m (b) 12 m (c) 24 m (d) 27 m
	8.3.4 An isosceles triangle has two sides of length 21 cm and a base of 26 cm. The base of a similar triangle is 24 cm. Find the perimeter of the smaller triangle.
8.4 Solve problems involving percents.	8.4.1 At a pre-season sale, a $200 coat was sold for $150. What is the percent of discount? (a) 30% (b) 25% (c) 20% (d) 15%
	8.4.2 A sporting goods store purchases Air Jordan athletic shoes at $30 per pair and marks them up 300%. What is the selling price of the shoes?

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 9: The learner will explore, graph, and interpret nonlinear equations.

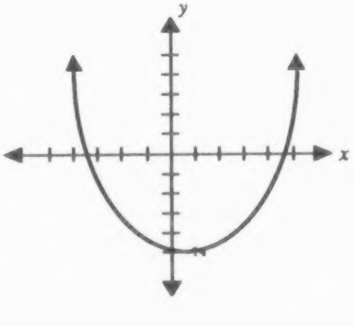
OBJECTIVES	SAMPLE MEASURES
9.1 Graph a quadratic equation.	9.1.1 Which is the graph of $y = x^2$?     9.1.2 Draw the graph of $y = x^2 - x - 6$ by a method of your choice. If you have access to a computer or calculator with graphing capabilities, use it to verify your answer.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 9: The learner will explore, graph, and interpret nonlinear equations.

OBJECTIVES	SAMPLE MEASURES
9.2 Use an automatic grapher to find the solution to a quadratic equation.	9.2.1 Use an automatic grapher to solve $4x^2 = 5 - 8x$. (a) { -2.5, 0.5} (b) {0.5, 2.5 } (c) { -0.5, 2.5} (d) { -2.5, -0.5}
	9.2.2 Use an automatic grapher to solve $1.1x^2 + 3.2x = 1.2$
	9.2.3 Use the graph to approximate the solution of $x^2 - x - 12 = 2$. 

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 9: The learner will explore, graph, and interpret nonlinear equations.

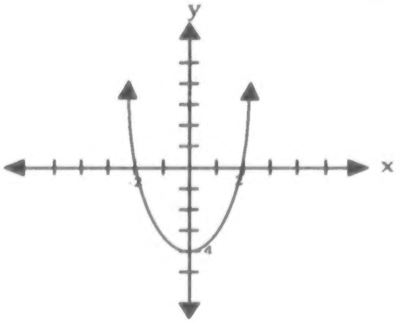
OBJECTIVES	SAMPLE MEASURES
9.3 Solve a quadratic equation when one member is in factored form and the other member is zero.	9.3.1 Solve $(2x - 1)(x - 3) = 0$. (a) $\{2, 3\}$ (b) $\{2, -3\}$ (c) $\{-2, 3\}$ (d) $\{\frac{1}{2}, 3\}$
	9.3..2 Explain how to solve $(x + 4)(x - 5) = 0$
9.4 Solve a second degree equation by factoring.	9.4.1 Solve $x^2 - x - 12 = 0$ (a) $\{-3, 4\}$ (b) $\{3, -4\}$ (c) $\{0, 3\}$ (d) $\{0, 4\}$
	9.4.2 Solve $a^2 = 5a + 6$
9.5 Use an automatic grapher to relate the solutions of quadratic equations and the x-intercepts.	9.5.1 Use an automatic grapher to find the x-intercepts of $y = x^2 + x - 12$. Solve the equation $x^2 + x - 12 = 0$ by factoring. How are your answers to these two questions related? Why is this reasonable?

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 9: The learner will explore, graph, and interpret nonlinear equations.

OBJECTIVES	SAMPLE MEASURES
9.6 Understand that the vertex provides the maximum or minimum value of a function.	9.6.1 The graph of $y = x^2 - 4$ is shown below. What is the smallest y-value on the graph? (a) - 2 (b) 2 (c) - 4 (d) 0 (e) - 6  9.6.2 When a ball is thrown into the air, its height depends on the time since the ball was thrown and an equation for its height is $h = -t^2 + 2t + 3$. Use an automatic grapher to find the greatest height above the ground attained by the ball. When does the ball hit the ground?

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 9: The learner will explore, graph, and interpret nonlinear equations.

OBJECTIVES	SAMPLE MEASURES
9.7 Solve a quadratic equation in which a perfect square equals a constant.	9.7.1 Solve: $x^2 = 25$ (a) $\{5\}$ (b) $\{-5, 5\}$ (c) $\{25, 1\}$ (d) $\{-25, 1\}$ 9.7.2 Solve: $(x-3)^2 = 16$.
9.8 Solve a quadratic equation by using the quadratic formula.	9.8.1 Solve: $x^2 - 10x - 20 = 0$ (a) $\{-5+3\sqrt{5}, -5-3\sqrt{5}\}$ (b) $\{3\sqrt{5}+5, 3\sqrt{5}-5\}$ (c) $\{5+3\sqrt{5}, 5-3\sqrt{5}\}$ (d) $\{-3\sqrt{5}+5, -3\sqrt{5}-5\}$ 9.8.2 Kyle has a rectangular garden that measures 12m by 14m. He wants to increase the area to $255m^2$ by increasing the width and length by the same amount. What will be the dimensions of the new garden?
9.9 Use quadratic equations to solve problems.	9.9.1 The sum of a number and its square is 72. Find the numbers. (a) -9, 8 (b) -8, 9 (c) 9 (d) 8 9.9.2 Use an automatic grapher to solve: $1.4x^2 - 0.7x = 0.2$

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 9: The learner will explore, graph, and interpret nonlinear equations.

OBJECTIVES	SAMPLE MEASURES

9.10 Determine if a set of data represents an exponential function.

9.10.1 Which of the following tables of data exhibit exponential behavior?

(a)

x	y
0	3
5	6
10	9
15	12
20	15
25	18

(b)

x	y
1	3.1
2	3.9
3	5.0
4	6.2
5	7.8
6	10

(c)

x	y
0	2
5	8
10	18
15	32
20	50
25	72

(d)

x	y
0	400
5	200
10	100
15	50
20	25
25	12.5

9.10.2 Explain to a friend how to test a set of data to determine if it represents exponential behavior.

MATHEMATICS

Algebra I

GRADE LEVEL 9-12

COMPETENCY GOAL 9: The learner will explore, graph, and interpret nonlinear equations.

OBJECTIVES	SAMPLE MEASURES
9.11 Use formulas, calculators and automatic graphers to explore and solve problems involving exponentials.	9.11.1 Evaluate to the nearest whole number: $A(1 + r)^x$ for $A = 120$, $r = .06$, and $x = 20$. (a) 103 (b) 525 (c) 385 (d) 2100
	9.11.2 Suppose that Geneva places $400 in a savings account that earns 8.5% interest compounded annually. How much money will she have in the account in 12 years? (a) $434.00 (b) $642,866.41 (c) $740.00 (d) $1,064.67
	9.11.3 Twenty years ago Juan's parents put some money in an account that has earned 7.8% interest compounded annually. He now has $2,357.96 in the account. How much did his parents place in the account initially?
	9.11.4 Translate into an expression: An initial population of 52 small animals is growing at the zoo at a rate of 25% a year. How many animals will there be after five years? (a) $52(1.25)^5$ (b) $52(1-.25)^5$ (c) $52(.25)^5$ (d) $(52\cdot1.25)^5$

NOTES

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will use and read measuring devices and solve problems involving the customary and metric units of measurement.

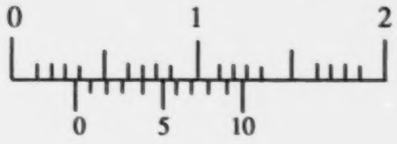
OBJECTIVES		SAMPLE MEASURES	
1.1	Find the mass/weight of objects using customary and metric units.	1.1.1	Assume water weighs 8.3 lb/gal What is the weight of a 100-gal tank filled with water if the empty tank weighs 20 pounds? (a) 83 lb (b) 832 lb (c) 830 lb (d) 850 lb
		1.1.2	If the average paper clip has a mass of 1 gram and a standard box holds 100 paper clips, how many boxes would be needed to have a mass of 1 kg?
1.2	Find the volume/capacity of quantities using customary and metric units.	1.2.1	To make lemonade, empty a 6-ounce can of frozen lemonade into a pitcher and add three cans of cold water. If you double this recipe, what is the smallest size pitcher that would hold the lemonade mixture? (a) pint (b) half-gallon (c) quart (d) gallon
		1.2.2	If a can of soft drink holds 355 mL, how many cans would be needed to fill twenty cups which hold 50 mL each?
1.3	Compare and convert measurement units as needed to perform calculations, using a calculator when needed.	1.3.1	How many meters are in 12.4 kilometers? (a) .124m (b) 124 m (c) 1,240 m (d) 12,400 m
		1.3.2	Which is larger, a liter or a quart?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will use and read measuring devices and solve problems involving the customary and metric units of measurement.

OBJECTIVES	SAMPLE MEASURES
1.4 Use common measuring tools and solve problems involving the measurements.	1.4.1 The measurement shown below, taken with a vernier caliper to the nearest hundredth of a centimeter, is:  (a) .03 (b) .04 (c) .39 (d) 1.32
	1.4.2 If the outside diameter of a pipe is 4.76 cm and the inside diameter is 3.82 cm, how thick is the pipe?
1.5 Compare measurements to specified tolerances.	1.5.1 The specifications for the design of a piece of machinery show a length of 14.625 inches with a tolerance of ± 0.0005 inch. Which of the following is not an acceptable measurement? (a) 14.622 (b) 14.624 (c) 14.628 (d) 14.631
	1.5.2 What is the lower limit of a length shown as 1.537 inch ± 0.0008 inch?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will use and read measuring devices and solve problems involving the customary and metric units of measurement.

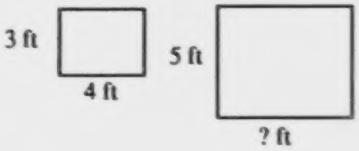
OBJECTIVES	SAMPLE MEASURES
1.6 Use significant digits to indicate accuracy of measurement.	1.6.1 Use a calculator to find the sum of the following measurements with the correct number of significant digits: 12.6 meters, 0.67 meters, 567.3 meters, and 7 987.3 meters. (a) 588.5573 (b) 588.56 (c) 588.557 (d) 588.6
	1.6.2 The area of a rectangular room measuring 2.78m by 3.52m is computed and reported with the correct number of significant digits. How many decimal places would there be in the answer?
1.7 Select the appropriate measuring tool and units of measure to solve a problem.	1.7.1 Jesse is planning to paint the living room and bedroom walls in his house and needs to find the area of the surfaces to be painted. How could he determine the cost of painting these rooms?
	1.7.2 Doris needs to measure the thickness of a sheet of thin plastic which is thinner than a dime. What measuring device should she use to determine this?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will use ratios, proportions and percents to solve problems.

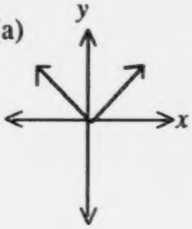
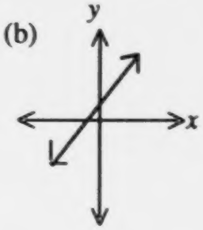
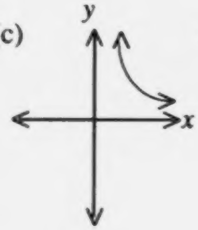
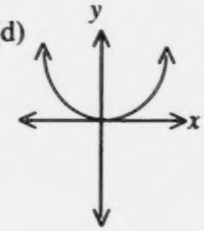
OBJECTIVES	SAMPLE MEASURES
2.1 Read, interpret and compare ratios.	2.1.1 Based on the statistic that "the Braves won five of the last six games", what is the win/loss ratio for those six games? (a) 5:1 (b) 5:6 (c) 1:5 (d) 5:11
	2.1.2 Write the ratio of 2 inches to 1.5 feet in four different ways.
2.2 Recognize and write proportions from given information.	2.2.1 Which of the following is not a true proportion? (a) $\frac{10}{35} = \frac{2}{7}$ (b) $\frac{14}{28} = \frac{4}{8}$ (c) $\frac{3}{5} = \frac{12}{20}$ (d) $\frac{4\text{in.}}{6\text{lb}} = \frac{2\text{in.}}{12\text{lb}}$
	2.2.2 Set up a proportion based on the following similar figures: 

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will use ratios, proportions and percents to solve problems.

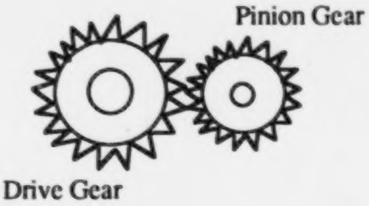
OBJECTIVES	SAMPLE MEASURES
2.3 Distinguish between direct and indirect relationships.	2.3.1 Which of the following graphs represents an indirect relationship?
	(a) 
	(b) 
	(c) 
	(d) 
	2.3.2 A person 5'6" tall casts a shadow 7' long. A flagpole 192'6" tall casts a shadow 245' long. How can you determine if there is a direct or indirect relationship between the height of an object and the length of its shadow?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will use ratios, proportions and percents to solve problems.

OBJECTIVES	SAMPLE MEASURES
2.4 Solve problems involving proportions.	2.4.1 If you are traveling at 50 mph you will travel 50 miles in 1 hour. If you travel for 3 hours at 50 mph, how far will you travel? (a) $16\frac{2}{3}$ mi (b) 150 mi (c) 100 mi (d) 300 mi
	2.4.2 The angular speed of each moving gear is inversely proportional to the number of teeth on the gear. Suppose the drive gear has 60 teeth and the pinion gear has 20 teeth. If the drive gear makes 10 revolutions per minute, how many revolutions per minute will the pinion gear make? 
	2.4.3 If 0.5 pints of oil is mixed with 2 gal of gasoline for a 2-cycle gasoline engine, how much oil should be mixed with 1.5 gal?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will use ratios, proportions and percents to solve problems.

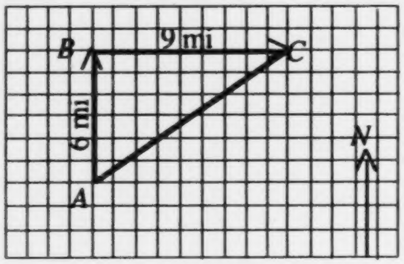
OBJECTIVES	SAMPLE MEASURES
2.5 Solve problems involving percents.	2.5.1 If you must score 70% on a certification exam to be licensed in a particular profession and you miss 50 out of 350 questions, what percent did you get correct? (a) 14% (b) 86% (c) 71% (d) 20% Did you pass the exam?
	2.5.2 A hairstylist earns $200 per week plus 40% of the work and sales income that she brings into the shop. If her work and sales for one week bring in $50 to the shop, what is her gross income that week?
	2.5.3 One store sells a micrometer for one-third off its usual price of $59.40. Another store sells the same micrometer for 25% off its usual price of $56.00. Which is the better buy and by how much?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will use ratios, proportions and percents to solve problems.

OBJECTIVES	SAMPLE MEASURES
2.6 Interpret and construct maps and scale drawings.	2.6.1 Locate a road map of the United States to determine what city is approximately 400 miles from High Point, North Carolina (a) Atlanta (b) Nashville (c) Chicago (d) New York City
	2.6.2 You want to frame a picture that is 11" x 14". There will be a mat border around it that is 1.5 inches wide. Make a $\frac{1}{4}$ scale drawing of the picture with the mat border.
2.7 Find the magnitude and direction of a vector.	2.7.1 What is the direction of vector *BC*?  (a) South (b) West (c) North (d) East
	2.7.2 What is the direction and length in miles of vector *AC* in 2.7.1?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will use ratios, proportions and percents to solve problems.

OBJECTIVES	SAMPLE MEASURES
2.8 Solve problems using signed numbers and vectors.	2.8.1 The altitude of Owens Telescope Peak in southern California is 11,045 feet above sea level. Just a few miles away, the floor of Death Valley is 282 feet below sea level. How many feet of altitude would a helicopter pilot have to climb to rise from the floor of Death Valley to the top of Owens Telescope Peak? (a) 10,763 ft (b) 10,863 ft (c) 11,227 ft (d) 11,327 ft
	2.8.2 Construct the vector described: The ground speed and direction of an airplane flying east with an indicated air speed of 125 miles per hour in a head wind of 15 miles per hour from the east.

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will solve geometric problems involving two- and three-dimensional shapes.

OBJECTIVES	SAMPLE MEASURES
3.1 Identify polygons and their properties.	3.1.1 Draw and describe the following triangles: (a) right (b) scalene (c) isosceles (d) obtuse
	3.1.2 Diagonals are often measured to check the construction of a rectangle. Describe briefly how this information is used.
3.2 Calculate the perimeter, area, and circumference of polygons and circles.	3.2.1 A trapezoidal table top for special use in a reading classroom measures 60 cm on the shorter base and 90 cm on the longer base. If the height is 50 cm, how much Formica is needed to re-cover the top?
	3.2.2 A car tire has a diameter of 23". How far will the car tire move if it makes a complete revolution?
	3.2.3 A circle is to be cut out of a square piece of metal 45 cm on a side. What is the least amount of metal that can be wasted?
3.3 Identify common three-dimensional shapes.	3.3.1 Give three examples of common objects shaped as spheres.
	3.3.2 Give three examples of common objects which are cylindrically shaped.
3.4 Calculate the surface area and the volume of three-dimensional shapes.	3.4.1 How many square feet of plastic material are needed to devise a wind tunnel cone whose base is 3 ft across and whose height is 4 ft with a 5-ft slant height?
	3.4.2 How many cubic inches are in an aluminum can $2\frac{1}{2}$" in diameter and $4\frac{3}{4}$" tall? (Round to tenths)

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will solve geometric problems involving two- and three-dimensional shapes.

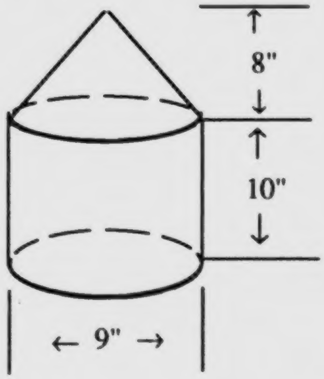
OBJECTIVES	SAMPLE MEASURES
3.5 Solve problems involving common geometric properties.	3.5.1 If there are about 250 cubic feet in each ton of baied alfalfa, how many tons of baled alfalfa can be stored in a 20 ft x 40 ft x 10 ft rectangular storage area? (a) 8000 tons (b) 8250 tons (c) 7750 tons (d) 32 tons
	3.5.2 There are 28 cylindrical columns in a shopping mall that need repainting. Each column measures 14 ft tall and 4.7 ft around. If a gallon of paint covers 400 square feet, how many gallons are needed to paint all 28 columns?
	3.5.3 The largest ground area covered by an office building is covered by the Pentagon in Arlington, Virginia. The ground area is a regular pentagon and each side is 921 ft. This pentagon can be divided into five congruent isosceles triangles of height 633 ft. Find the ground area.
3.6 Draw auxiliary diagrams to help solve for an unknown dimension or an unknown angle in both plane and solid shapes.	3.6.1 To make a rectangular table more stable, a diagonal brace is attached to the underside of the table surface. Draw a diagram to represent a table 27 dm by 36 dm. Then find the length of the brace.
	3.6.2 Draw a diagram of a right circular tank 6' high and 12' in diameter. The water level is at 3' 6". How many more cubic feet of water can the tank hold?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will solve geometric problems involving two- and three-dimensional shapes.

OBJECTIVES	SAMPLE MEASURES
3.7 Solve both plane and solid geometry problems that involve a series of successive calculations.	3.7.1 Find the weight of a cast iron solid as shown in the diagram if cast iron weighs 0.26 lb per cubic inch. (Round to nearest pound.) 
	3.7.2 The lampshades in 50 rooms in a hotel must be replaced by recovering the outside of each with new material. The lampshades are shaped like right circular cylinders 12" in diameter and 16" tall. Each room has two lamp shades. Approximately how many square yards of fabric are needed to complete the job?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will analyze patterns and functions.

OBJECTIVES	SAMPLE MEASURES
4.1 Decode and extend patterns.	4.1.1 Find the pattern in the following numbers and generate the next three numbers in the sequence: 1, 1, 2, 3, 5, 8, 13, __, __, __ .
	4.1.2 The table below shows the relationship between the time it takes sound to travel and the distance sound travels. Describe this relationship.

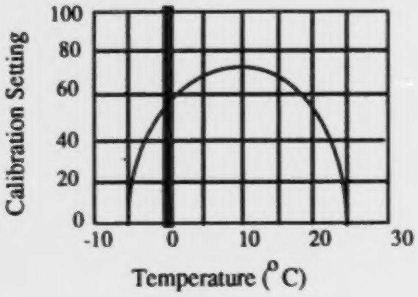
Time in seconds	Distance in kilometers
3	1
6	2
9	3
12	4
. . .	. . .

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will analyze patterns and functions.

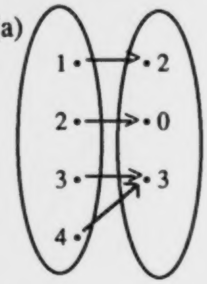
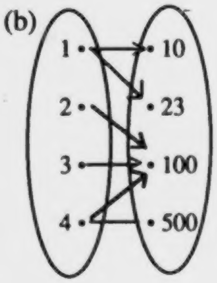
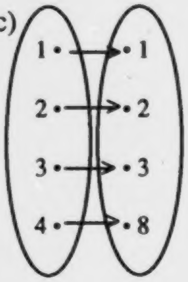
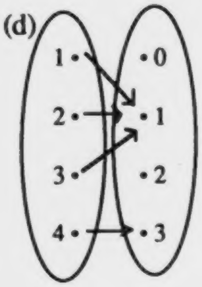
OBJECTIVES	SAMPLE MEASURES
4.2 Identify the domain and range of mathematical relations and functions.	4.2.1 Given the equation $p = 4s$ for finding the perimeter of a square, what is the domain of this function? (a) all real numbers (b) all integers (c) all positive real numbers (d) all positive integers
	4.2.2 The graph below represents an equation, $c = (-0.25T^2 + 5T + 35)$, for calibrating an electronic instrument. Describe the domain and range of this graph. 

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will analyze patterns and functions.

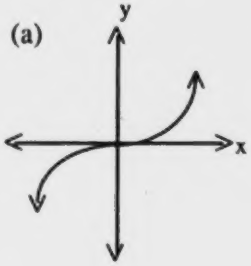
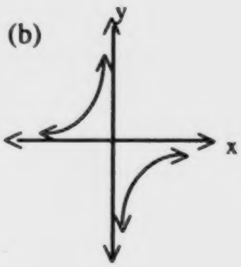
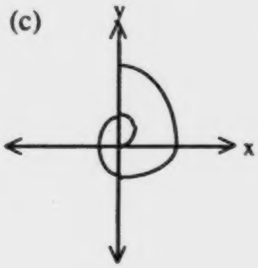
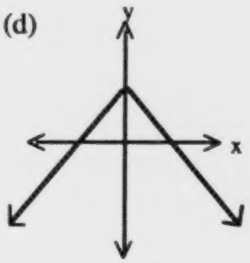
OBJECTIVES	SAMPLE MEASURES
4.3 Distinguish between mathematical relations and functions.	4.3.1 Given the mappings of the relations illustrated below, identify which ARE functions, and which ARE NOT. (a)  (b)  1 2 3 4 · 10 23 100 500 (c)  (d) 

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will analyze patterns and functions.

OBJECTIVES	SAMPLE MEASURES
4.3 Distinguish between mathematical relations and functions.	4.3.2 Use the "pencil test" or other means, to identify which of the graphs below represents functions of the form $y = f(x)$, and which do not. (a)  (b)  (c)  (d)  4.3.3 A function must: (a) produce one and only one output value for each input value. (b) have a value defined for $x = 0$. (c) not have the variable x raised to a power higher than 2. (d) have a graph that crosses the x-axis.

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will analyze patterns and functions.

OBJECTIVES	SAMPLE MEASURES
4.3 Distinguish between mathematical relations and functions.	4.3.4 Explain in writing whether the information in the table below represents a relation or a function.

NR Insurance Company

Age (years)	Category
0 - 1	A
2 - 5	B
6 - 12	C
13 - 17	D
18 - 25	E or F
26 - 39	G
40 - 58	H
59 & above	J

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will analyze patterns and functions.

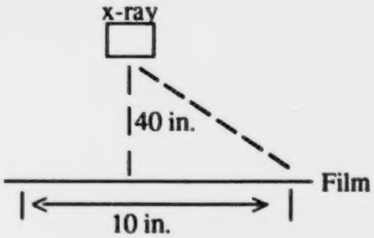
<table>
<tr><th>OBJECTIVES</th><th>SAMPLE MEASURES</th></tr>
<tr><td>4.4 Use patterns and functions to solve problems at home and at work.</td><td>4.4.1 On New Year's Day you begin saving pennies in a large jar. On the first day (January 1) you place one penny in the jar. On the second day, you add two pennies. On the third day, you add three pennies, and so on. On which day will you have 25 cents in the jar?
4.4.2 You question the accuracy of a new outdoor thermometer, so you compare the weather reports with your new thermometer's readings. The results are summarized in the table below. Write a statement or an equation that shows the relationship between the weather station's temperature and the new thermometer's reading.
<table>
<tr><th>Weather Station</th><th>New Thermometer</th></tr>
<tr><td>48°</td><td>43°</td></tr>
<tr><td>81°</td><td>76°</td></tr>
<tr><td>59°</td><td>54°</td></tr>
<tr><td>65°</td><td>60°</td></tr>
<tr><td>74°</td><td>69°</td></tr>
</table>
4.4.3 A contractor gets quotes from two different well diggers. Wally's Well Digging charges $200 + $5 per foot. Beaver's Well Digging charges $6 per foot. Discuss how the contactor should decide which well digger to use.</td></tr>
</table>

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will solve problems using triangle relationships.

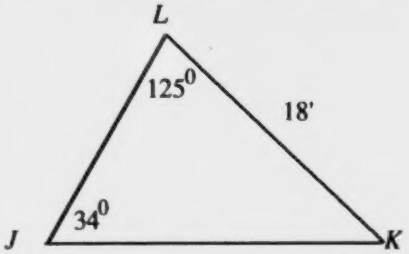
OBJECTIVES	SAMPLE MEASURES
5.1 Use the Pythagorean Theorem to find a side of a right triangle.	5.1.1 A light pole will be braced with one wire that is to be tied to a stake in the ground 18 feet from the base of the pole which extends 26 feet above the ground. If the wire is attached to the pole 2 feet from the top, how much wire must be used to brace the pole?
	5.1.2 An x-ray source is positioned over a piece of film as shown below. The source is centered 40 inches away from a piece of film that is 10 inches wide. How far must the x-rays travel to reach the edge of the film? 
5.2 Use the characteristics of 3:4:5, $45^0 - 45^0 - 90^0$ and $30^0 - 60^0 - 90^0$ right triangles to solve practical problems.	5.2.1 A rafter makes a 30^0 angle with the horizontal. If the rise is 9 ft, find the rafter length and the run to the nearest inch.
	5.2.2 An art display includes two sound-effect speakers. They need to be positioned on the wall so that when you stand midway between the speakers, you are no closer than 7 feet to either speaker. Each speaker should be the same distance from the corner. How far from the corner should you position each speaker?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will solve problems using triangle relationships.

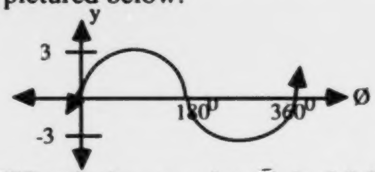
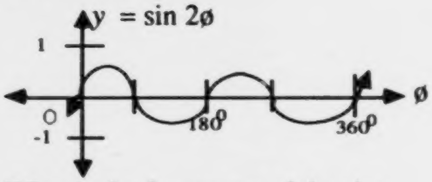
OBJECTIVES	SAMPLE MEASURES
5.3 Use the ratios for the sine, cosine, and tangent of an angle to solve problems that involve right triangles.	5.3.1 As you marvel at the height of a very tall broadcasting antenna tower, you decide to try to estimate its height. Closely watching your car's odometer, you drive exactly one-half mile from the tower's base. Then, using the protractor, you find the angle of elevation to be 20^0 above the horizon. Find the height of the tower.
	5.3.2 A stretch of roadway drops 30 feet for every 300 feet of road. Find the angle of declination of the road.
5.4 Solve problems involving the Law of Cosines and the Law of Sines.	5.4.1 A surveyor needs the measure of $\overline{JK}$ to the nearest foot. How could this be found? Solve the problem. 
	5.4.2 A triangular lot has sides 180 feet long, 160 feet long and 123.5 feet long. Find the angles of the corners of the lot.

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will graph and analyze trigonometric functions.

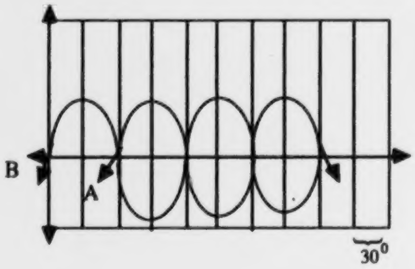
	OBJECTIVES		SAMPLE MEASURES
6.1	Draw a graph of sine and cosine waves.	6.1.1	Graph a sine curve over the interval. $0^0 \leq \O \leq 360^0 \quad y = \sin \O$.
		6.1.2	Graph a cosine curve over the interval. $0^0 \leq \O \leq 360^0 \quad y = \cos \O$.
		6.1.3	Using a graphics calculator, graph $y = \sin \O$ and $y = \cos \O$. State how the graphs are alike and how they are different.
		6.1.4	The standard electric voltage in a 60-Hz alternating current circuit is given by $V = 170 \sin 120 \pi t$, where t is the time in seconds. Sketch the graph of V as a function of t for $0 \leq t \leq .05$s.
6.2	Find the amplitude, wave length, period, and frequency of sine waves.	6.2.1	State the amplitude of $y = 3\sin\O$.
		6.2.2	Write an equation of the sine curve pictured below: 
		6.2.3	What is the wave length in 6.2.2?
		6.2.4	What period is shown in the graph below? 
		6.2.5	What is the frequency of the sine wave in 6.2.4?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will graph and analyze trigonometric functions.

OBJECTIVES	SAMPLE MEASURES
6.3 Find the phase shift between two sine waves.	6.3.1 Using the graph below, state the phase shift difference between A and B.  6.3.2 Using a graphics calculator, show the graphs of $y = 3 \cos (\varnothing - 30^0)$ and $y = 3 \cos (\varnothing + 30^0)$ on the same display. What do you notice?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will use formulas to solve problems.

OBJECTIVES	SAMPLE MEASURES
7.1 Translate a problem into an equation.	7.1.1 Translate into an equation: Olga earns an 8% commission on sales plus a base salary of $1200 per month. Last month, she earned a total of $2500. What were her total sales?
	7.1.2 Write an equation which could be used to solve: Find the dimensions of a rectangular cover plate if its length is 6 inches longer than its width and its perimeter is 68 inches.
7.2 Evaluate algebraic expressions and formulas when replacement values are given.	7.2.1 Evaluate $a^2 - 2ab + c^2$ when $a = -1$ $b = 2$, and $c = -5$.
	7.2.2 Use the formula $V = \pi r^2 h$ to solve: A natural gas company runs an 8" pipeline (8" inside diameter) to one subdivision of houses that is 10 miles away. How many cubic feet of natural gas are stored in the pipeline?
7.3 Select an appropriate formula to solve a problem.	7.3.1 Which formula could best be used to find the area of a circular solar cell which has a 6 cm diameter? Use this formula to solve the problem: (a) $2\pi r$ (b) $\frac{\pi d^2}{4}$ (c) πd (d) None of these
	7.3.2 Use a formula to help you solve the following: A ladder 8 m long is placed 4.8 m from a wall. How far up the wall does the ladder reach?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will use formulas to solve problems.

OBJECTIVES		SAMPLE MEASURES	
7.4	Solve problems with formulas using a calculator when needed.	7.4.1	Find the time in years if the principal invested is $500, the annual interest rate is 9% and the simple interest earned is $135. ($i = prt$)
		7.4.2	Use a calculator to estimate the population of the United States in the year 2025 using the formula: U. S. population (in millions) $75.9 \times 2^{\frac{(y-1900)}{50}}$
		7.4.3	A jet engine developing T lb of thrust and driving an airplane at V mph, has a thrust horsepower, H, given approximately by the formula $H = \frac{TV}{375}$. Find the airspeed V if $H = 16{,}000$ hp and $T = 10{,}000$ lb.
		7.4.4	Suppose fire and smoke damage your house, causing a loss totaling $8750. Your insurance policy provides for 80% co-payment, using the formula: $P = \frac{IL}{0.8V}$ where P is the payment made by the insurance company, I is the insurance coverage of the policy, L is the loss due to fire, and, V is the value of the property that is insured. Determine the payment you should receive from the insurance company if your property is valued at $65,000 and the insurance policy covers $50,000.

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will use formulas to solve problems.

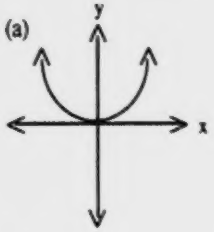
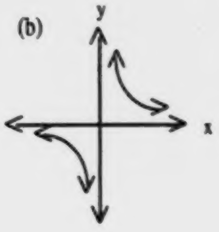
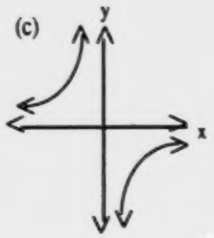
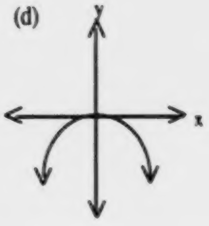
OBJECTIVES	SAMPLE MEASURES
7.5 Solve problems that involve powers and roots.	7.5.1 Find the power used in an electric light bulb, $P = i^2R$, if the current $i = 0.8$ ampere and the resistance $R = 150$ ohms. P will be in watts.
	7.5.2 If the horsepower rating of an engine is given by: $HP = \frac{D^2N}{2.5}$ find D(the bore in inches) if the number of cylinders (N) is 6 and the horsepower rating is 200.
	7.5.3 The manufacturer of a new line of electronic equipment expects the projected revenue R for the next twelve months to be given by the formula $R = 610{,}000 \times (1.56)^{\frac{1}{4}t}$ where R is the projected revenue and t is the number of months from 0 to 12. What revenue is the manufacturer projecting after the sixth month? (Round to the nearest $1000.)
	7.5.4 At an inflation rate of 6% a year, the cost of a $12,000 car will increase to approximately $12{,}000\,(1.06)^3$ in 3 years. What will the approximate cost be then?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will solve problems that involve nonlinear equations.

OBJECTIVES	SAMPLE MEASURES
8.1 Recognize nonlinear equations (involving squares, square roots and reciprocals of the variable) and become familiar with their graphs.	8.1.1 Which of the following equations represent a linear relationship and which equations represent a nonlinear relationship? (a) $d = rt$ (b) $A = \pi r^2$ (c) $y = \sqrt{x}$ (d) $V = \frac{4\pi r^3}{3}$
	8.1.2 Which of the following represents a graph $y = 1/x$? (a)  (b)  (c)  (d) 

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will solve problems that involve nonlinear equations.

OBJECTIVES		SAMPLE MEASURES	
8.2	Solve and graph a nonlinear equation.	8.2.1	The formula for compund interest is $A = P(1 + r)^n$ where A is the amount after n interest periods; P is the principal, and r is the interest rate. A business invested $40,000 in an account which paid 6% interest compounded quarterly. How much money would be in the account after four years? Draw a graph to show the relationship between the time and the amount over this four-year period.
		8.2.2	The horsepower of an engine is given by: $HP = \frac{D^2 N}{2.5}$ where D is the bore of the cylinder in inches, N is the number of cylinders. Provide an appropriate graph for horsepower of a 6-cylinder engine.

MATHEMATICS

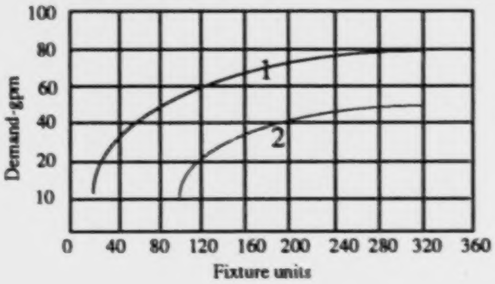
Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will solve problems that involve nonlinear equations.

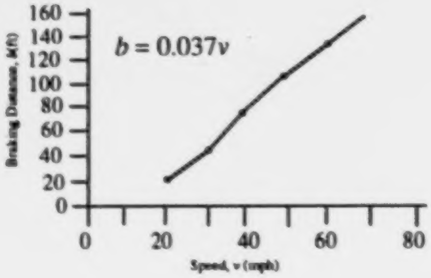
OBJECTIVES	SAMPLE MEASURES
8.3 Read values from the graph of a nonlinear equation.	8.3.1 From the graph below determine the difference in gallons per minute for 140 flush valve units compared to 140 tank valve units.

Demand Load Curves for Water Systems



#1 represents a system predominantly of flush valves.
#2 represents a system predominantly of tank valves.

8.3.2 From the graph determine the braking distance of an automobile traveling at 50 mph.



MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will solve problems involving systems of equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
9.1 Write equations and inequalities in terms of two unknowns.	9.1.1 Write the system of equations that result from the following problem using the chart below: A materials yard wishes to make a 900 cubic foot mixture of two different types of soil. One type of soil is 10% phosphorous and the other is 25% phosphorous. How much of each type of soil should be mixed in order to obtain a soil that is 15% phosphorous?

	Soil A	Soil B	Total
Amt.(ft^3)	A	B	900
% Phos.	10	25	15
ft^3 Phos.	.10A	.25B	135

9.1.2 A refinery produces gasoline and diesel fuel. The total production is at most 2000 gallons per day. Based upon past sales, it should produce at least twice as much gasoline as diesel fuel. What are the different production combinations of the two fuels?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will solve problems involving systems of equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
9.2 Solve systems of equations.	9.2.1 In framing a wall of an office building, each horizontal plate (top and bottom) is 3 ft 9 in. longer than each vertical stud. Find the lenghts of each plate and each of the nine studs if a total of 81 ft 9 in. of lumber is used.
	9.2.2 The velocity of sound in steel is 449ft/s faster than the velocity of sound in air. One end of a long steel bar is struck and an observer at the other end measures the time it takes for the sound to reach him. He finds that the sound through the bar takes .012*s* to reach him and that the sound through the air takes .018*s*. What are the velocities of sound in air and in steel?
	9.2.3 The distance S that a crate moves down an incline chute is given by $S = V_0 t + \frac{1}{2}at^2$, where V^0 is the initial velocity, a is the acceleration, and t is the time of descent. Find V and a if $S = 3.8$m for $t = 4.5s$ and $S = 6.5$m for $t = 6$ s.

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will solve problems involving systems of equations and inequalities.

OBJECTIVES		SAMPLE MEASURES	
9.3	Solve problems involving inequalities.	9.3.1	An investor wishes to invest no more than \$20,000 for five years. A very safe investment should yield a 3.8% average return while a fairly risky investment should yield an 7.2% average return. The investor has two requirements: the return should exceed \$5000 over the five-year period and the amount invested at risk should be less than half the amount invested safely. Define the possible combinations of solutions graphically. Write a note to the investor explaining your advice to her based upon this information.
		9.3.2	The current value of a shredder in a scrapyard is determined by the formula $V = P - td$ where P is the original price, t is the number of years, and d is the yearly depreciation. If the original price was \$84,500, what yearly depreciation is needed in order to make the current value become between \$30,000 and \$40,000 in 7 years?
		9.3.3	The object distance p and image distance g for a camera of focal length 3.00 cm is given by $p = \frac{3.00g}{(g - 3.00)}$. For what values of g is $p > 12.0$ cm?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will solve problems involving systems of equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
9.4 Solve linear programming problems.	9.4.1 A telephone company installs two types of cable. It estimates that no more than 300m of type A cable and at least 200m but no more than 400m of type B cable are needed. Graph the possible lengths of cables that are needed.
	9.4.2 Use the following linear programming problem to answer the questions below: Constraints: $5x + 10y \leq 58$ $10x + 12y \leq 101$ $x \geq 4$ Maximize: $P = 1.44x + 2.14y$ a. Graph the feasible region. b. Which points are in the feasible region? (5, 1.92) (7, 1.95) (6, 3) (9, 0) c. Find the coordinate of the four corners of the feasible region and calculate P for each corner. d. Find the optimal solution.
	9.4.2 A manufacturer has two plastic molding shops and must produce at least 900 model planes and at least 1500 model cars each day. Shop A can make 60 planes and 120 cars per hour. Shop B can make 100 planes and 150 cars per hour. The cost of running the machine in shop A is \$2.55/h and the cost of running the machines in shop B is \$3.20/h. How many hours should each machine be used every day to minimize the cost?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 10: The learner will use statistics to analyze and solve real-world problems.

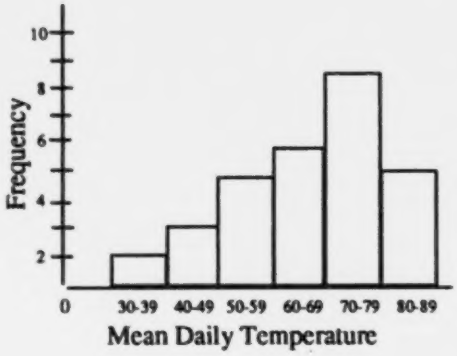
OBJECTIVES	SAMPLE MEASURES
10.1 Distinguish between mean, mode, and median as measures of central tendency.	10.1.1 In researching the salaries of individuals from Michael Jordan's high school graduating class, how would each measure of central tendency be influenced by his multi-million dollar contract with the Chicago Bulls?
	10.1.2 Which measure of central tendency is calculated by finding the arithmetic average? (a) mean (b) median (c) mode (d) standard deviation
10.2 Calculate the mean, mode, and median for a set of data.	10.2.1 Given the following test scores: 93, 99, 84, 97, 91, 77, 90, 89, 62, 73, and 68. Which is the median score? (a) 84 (b) 89 (c) 90 (d) 93
	10.2.2 What is the mean of the scores in 10.2.1?
	10.2.3 What is the mode of the scores in 10.2.1?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 10: The learner will use statistics to analyze and solve real-world problems.

OBJECTIVES	SAMPLE MEASURES
10.3 Draw and interpret a histogram to represent frequency distribution of data.	10.3.1 Create a histogram from the following data which were collected by the quality assurance division of a bottling plant with respect to the volume of cola in 2-liter bottles: 2.1, 2.1, 1.9, 1.8, 2.2, 2.2, 2.1, 2.1, 2.0, 1.8, 1.9, 1.9, 1.9, 2.1, 1.8, 1.9, 2.0, 2.1. 10.3.2 Based on the data displayed in the histogram, can you determine the true mean temperature for the month of June or just an approximation? Explain your answer. 

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 10: The learner will use statistics to analyze and solve real-world problems.

OBJECTIVES	SAMPLE MEASURES
10.4 Distinguish between range, trend and standard deviation as measures of variability.	10.4.1 Discuss the similarities and differences of the following measures of variability: range, trend, and standard deviation.
	10.4.2 Which measures of variability answer the following questions? (a) What was the span of grades made on the test? (b) Are the test grades improving as the year goes on? (c) Are most students making close to the same grade or are the grades really spread out?
10.5 Interpret the characteristics of a normal curve.	10.5.1 Frequencies of the data values are plotted along which axis of a normal curve? (a) x-axis (b) z-axis (c) y-axis (d) origin
	10.5.2 For a normal curve, what percent of the distribution is within one standard deviation of the mean? within two standard deviations of the mean? within three standard deviations of the mean?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 10: The learner will use statistics to analyze and solve real-world problems.

OBJECTIVES	SAMPLE MEASURES
10.6 Calculate the range and standard deviation to describe a set of data.	10.6.1 Given the following test scores: 91, 97, 84, 99, 93, 77, 80, 89, 62, 73, and 68. What is the range? What is the standard deviation?
	10.6.2 A strobe light is designed to flash every 2.25 seconds at a certain setting. Sample bulbs were tested with the results displayed in the following table:

Number of bulbs	Time between flashes
2	2.21
7	2.22
18	2.23
41	2.24
56	2.25
32	2.26
8	2.27
3	2.28
3	2.29

Find the standard deviation for the distribution.

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 11: The learner will use probability to solve real-world problems.

OBJECTIVES	SAMPLE MEASURES
11.1 Find the probability of some simple events.	11.1.1 A telephone number in the United States has a 3-digit area code, a 3-digit exchange, and a 4-digit station code. Suppose someone dials your area code and your exchange, but accidentally reverses the last two digits of the number. What is the probability that the wrong number dialed is your number?
	11.1.2 A large piggy bank has 102 coins in it: 75 pennies, 9 nickels, 10 dimes, and 8 quarters. It is equally likely that any of the 102 coins will fall out when the bank is turned upside down and shaken. What is the probability that the coin shaken out will be a quarter?
11.2 Count the number of ways an event can happen.	11.2.1 A coach needs to fill seven positions on the baseball team. If 16 people try out, how many different ways can the coach choose seven players for the team?
	11.2.2 You need to schedule 8 employees to work on 3 shifts. Each shift requires 2 workers. How many different ways are there to schedule these workers to these shifts?

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 11: The learner will use probability to solve real-world problems.

OBJECTIVES	SAMPLE MEASURES
11.3 Draw diagrams and charts to help find the probability of an event.	11.3.1 A white flower (with genes WW) is crossed with a pink flower (with genes RW). Make a chart to show the possible color combinations of the offspring from this cross. 11.3.2 A catalog advertises four different models of refrigerators. If each is available in three different colors, how many different orders can be placed for a refrigerator? Solve this problem by making a tree diagram and by using the multiplication rule.

MATHEMATICS

Technical Mathematics

GRADE LEVEL: 9-12

COMPETENCY GOAL 12: The learner will solve problems using computer technology.

OBJECTIVES	SAMPLE MEASURES
12.1 Load and use simple spreadsheet templates to solve practical problems.	*NOTE: Student will demonstrate mastery of Objectives 12.1 - 12.5 by performing the designated tasks under observation.
12.2 Use an appropriate graphics program to produce bar graphs, circle graphs, and line graphs.	
12.3 Use an appropriate graphics program to graph linear and nonlinear functions.	
12.4 Use an appropriate graphics program to graph one or more curves and determine such characteristics as their slope, roots, and intersection points.	
12.5 Use an appropriate graphics program to determine the effect of varying parameters in an equation on the shape of the curve representing the graphed equation.	

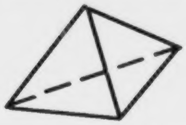
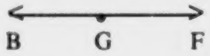
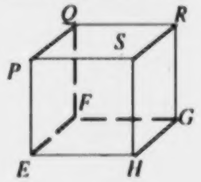
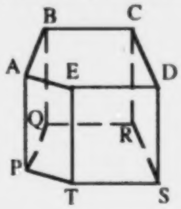
NOTES

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will use concepts of points, lines, and planes in one, two, and three dimensions.

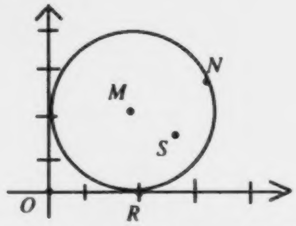
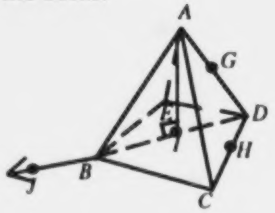
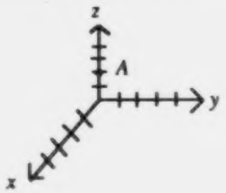
OBJECTIVES	SAMPLE MEASURES
1.1. Identify and name sets of points, such as line, ray, segment, and plane.	1.1.1 How many planes are shown?  (a) 1 (b) 2 (c) 3 (d) 4
	1.1.2 $\overrightarrow{BG} \cup \overrightarrow{FG} =$  (a) G (b) $\overrightarrow{BG}$ (c) $\overline{FG}$ (d) $\overleftrightarrow{BF}$
	1.1.3 Is $\overrightarrow{AB}$ the same as $\overrightarrow{BA}$? Explain.
	1.1.4 Name a fourth point in the same plane as *S*, *H*, and *G*. 
1.2 Draw representations of points, lines and planes.	1.2.1 Draw two intersecting planes. Name the intersection.
	1.2.2 Name the intersection of planes *BCRQ*, *CDSR*, and *ABCD*.  (a) *C* (b) *D* (c) $\overline{CR}$ (d) $\overline{BC}$

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will use concepts of points, lines, and planes in one, two, and three dimensions.

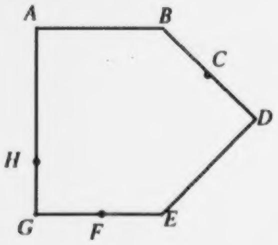
OBJECTIVES	SAMPLE MEASURES
1.3 Identify interior and exterior points in two-dimensional and three-dimensional figures.	1.3.1 Which of these points is exterior to circle *M*?  (a) *O* (b) *R* (c) *N* (d) *S* 1.3.2 Name the point in the interior of the solid. 
1.4 Find the coordinates of a point in a plane or in space.	1.4.1 The coordinates of *P* are (1, 3). If *P* is moved 4 units to the left, the coordinates of the new point are: (a) (1, 1) (b) (–3, 3) (c) (5, 3) (d) (–2, 3) 1.4.2 The coordinate of point *A* is ____. 

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will use concepts of points, lines, and planes in one, two, and three dimensions.

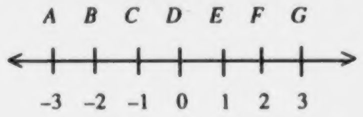
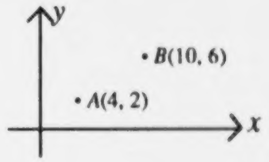
OBJECTIVES	SAMPLE MEASURES
1.5 Find the length of a segment on a line, in a plane, or in space.	1.5.1 What is the distance between (–4, –3) and (5, –7)? (a) $\sqrt{15}$ (b) $\sqrt{17}$ (c) $\sqrt{65}$ (d) $\sqrt{97}$ 1.5.2 Given: *A* (–1, 3, 2) and *B* (4, 1, –4). Approximate the length of $\overline{AB}$ to the nearest tenth. 1.5.3 On a number line, the coordinate of *A* is –6, *B* is –2, *C* is 5, and *D* is 10. Select the most accurate statement. (a) $AC = 1$ (b) $BC = -7$ (c) $AD = 4$ (d) $BD = 12$
1.6 Apply the segment addition postulate (definition of betweenness).	1.6.1 If $AB + BC = AC$, explain what must be true for point *B*. 1.6.2 Which is the most accurate statement?  (a) $AG + GE = AE$ (b) $HG + GF = HF$ (c) $EF + FG = EG$ (d) $BD + DE = BE$

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will use concepts of points, lines, and planes in one, two, and three dimensions.

OBJECTIVES		SAMPLE MEASURES	
1.7	Use lengths to solve problems involving geometric probability.	1.7.1	A point, P, is picked at random on $\overline{AG}$. What is the probability that P is on $\overline{BD}$?  (a) 0 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 1
		1.7.2	M is the midpoint of $\overline{AB}$ and P is the midpoint of $\overline{AM}$. If a point on $\overline{AB}$ is picked at random, what is the probability that the point is on $\overline{PM}$?
1.8	Identify the midpoint of a given segment on a line, in a plane or in space.	1.8.1	The coordinates of the midpoint of $\overline{AB}$ are:  (a) (3, 2) (b) (2, 3) (c) (7, 4) (d) (5, 6)
		1.8.2	The coordinate of A and the coordinate of the midpoint M of $\overline{AB}$ are given as follows: $A = 13$, $M = -15$. Find the coordinate of B. (a) 43 (b) –43 (c) –2 (d) 28
		1.8.3	M is the midpoint of $\overline{XY}$, $XM = 8 - t$ and $MY = 5t - 10$. Explain how to find the value of t.
		1.8.4	Find the length and midpoint of $\overline{AB}$, given A (3, 4, –1), B (6, –3, 2).

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will write a valid proof using a variety of reasoning strategies.

OBJECTIVES	SAMPLE MEASURES
2.1 Identify the structure of geometric deductive reasoning (undefined term, postulates, theorems).	2.1.1 In Euclidean geometry, which term below would have a definition? (a) Line (b) Point (c) Plane (d) Ray
	2.1.2 Definitions may be used to justify a statement. Name two other types of justifications.
2.2 State and use properties of equality and inequality.	2.2.1 Identify the property of equality illustrated below: If $2x - 3 = 27$, then $2x = 30$. (a) Multiplication (b) Addition (c) Distribution (d) Substitution
	2.2.2 Write a statement which illustrates the substitution property of equality.
2.3 Write and interpret statements in if-then form.	2.3.1 Choose the correct "if-then" form of this statement: "All rectangles are quadrilaterals." (a) If a figure is a rectangle, then it is a quadrilateral. (b) If a figure is a quadrilateral, then it is a rectangle. (c) Rectangles are quadrilaterals. (d) Some rectangles are quadrilaterals.
	2.3.2 Express the following statement in "if-then" form. "Equilateral triangles are equiangular."

*It is expected that proofs (flow diagram, two-column, paragraph and indirect) will be appropriately incorporated within each goal throughout the curriculum.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will write a valid proof using a variety of reasoning strategies.

OBJECTIVES	SAMPLE MEASURES
2.4 State the converse, inverse, and contrapositive of the conditional.	2.4.1 State the inverse of the following statement: "If x is not even, then $x + 1$ is not odd." (a) If $x + 1$ is odd, then x is even. (b) If $x + 1$ is not odd, then x is not even. (c) If x is even, then $x + 1$ is odd. (d) If $x + 1$ is even, then x is odd.
	2.4.2 Write the contrapositive of the following: If $AB + BC = AC$, then B is between A and C.
	2.4.3 Write the converse statement for: "If a ray bisects an angle, then the two angles formed are congruent."

*It is expected that proofs (flow diagram, two-column, paragraph and indirect) will be appropriately incorporated within each goal throughout the curriculum.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will write a valid proof using a variety of reasoning strategies.

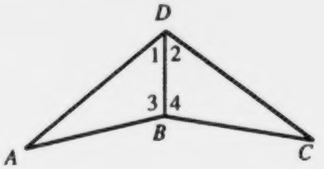
OBJECTIVES

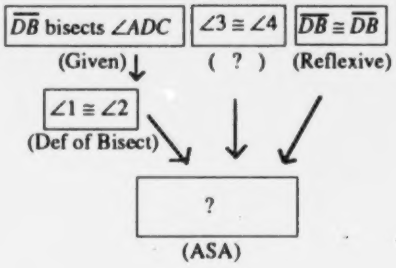
2.5 Write a proof using a flow diagram.

*It is expected that proofs (flow diagram, two-column, paragraph and indirect) will be appropriately incorporated within each goal throughout the curriculum.

SAMPLE MEASURES

2.5.1 Replace the (?) with the appropriate statement or reason. Given: $\overline{DB}$ bisects $\angle ADC$; $\angle 3 \cong \angle 4$, prove $\triangle ADB \cong \triangle CDB$.

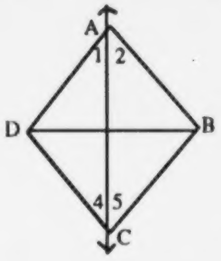



$\overline{DB}$ bisects $\angle ADC$
(Given)
$\angle 3 \cong \angle 4$
(?)
$\overline{DB} \cong \overline{DB}$
(Reflexive)
$\angle 1 \cong \angle 2$
(Def of Bisect)
?
(ASA)

2.5.2 Using a flow chart, prove the following:

Given: $\overline{AD} \perp \overline{AB}$
$\overline{CD} \perp \overline{BC}$
$\angle 1 \cong \angle 4$

Prove: $\angle 2 \cong \angle 5$.

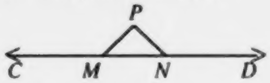
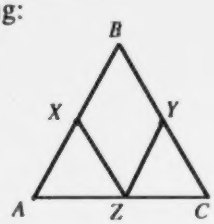
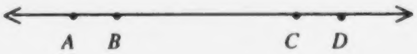
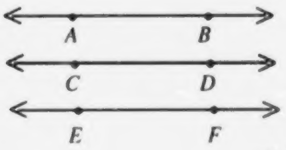


MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will write a valid proof using a variety of reasoning strategies.

OBJECTIVES	SAMPLE MEASURES
2.6 Write proofs using a two-column format.	2.6.1 Given: The figure with $\angle PMN \cong \angle PNM$ Prove: $\angle CMP \cong \angle DNP$ 
	2.6.2 Use a two-column format to prove the following: Given: $\overline{BX} \cong \overline{BY}$ $\overline{AX} \cong \overline{CY}$ Prove: $\angle A \cong \angle C$ 
2.7 Write proofs using a paragraph format.	2.7.1 Write a paragraph proof to show that two angles both equal in measure and supplementary each have a measure of 90^0.
	2.7.2 Write a paragraph proof to show that if $AB = CD$, then $AC = BD$. 
2.8 Write indirect proofs.	2.8.1 Write an indirect proof to show that a triangle cannot have two right angles.
	2.8.2 Write an indirect proof to show that two lines parallel to the same line are parallel to each other. 

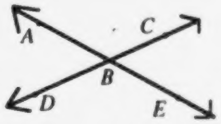
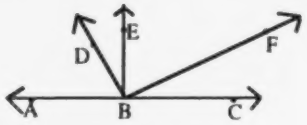
*It is expected that proofs (flow diagram, two-column, paragraph and indirect) will be appropriately incorporated within each goal throughout the curriculum.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will use properties of angles, lines, and planes to solve problems and write proofs.

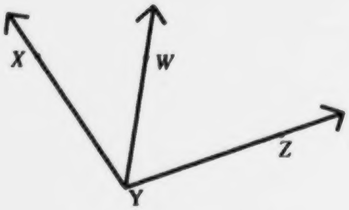
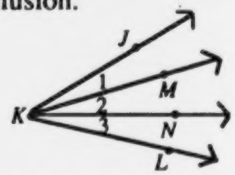
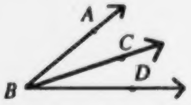
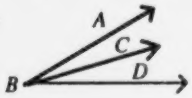
OBJECTIVES	SAMPLE MEASURES
3.1 Use the definitions of adjacent, vertical, linear pair, complementary and supplementary angles to solve problems and write proofs.	3.1.1 $m\angle ABD = 5x + 9$ and $m\angle CBE = 7x - 5$. Find $m\angle CBE$.  (a) 84 (b) 7 (c) 44 (d) 19 3.1.2 $\angle A$ and $\angle B$ are complementary. If $m\ \angle A = 5x - 6$ and $m\ \angle B = 3x$, find $m\ \angle A$. (a) 90 (b) 12 (c) 54 (d) 36 3.1.3 In this figure, $m\angle ABE = 90$, $m\angle DBE = 20$, $m\angle EBF = 70$, and $m\angle FBC = 20$. Name the angle congruent to $\angle EBF$. Explain why.  3.1.4 Draw a figure to disprove this statement: Adjacent angles are coplanar angles with a common side and a common vertex.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will use properties of angles, lines, and planes to solve problems and write proofs.

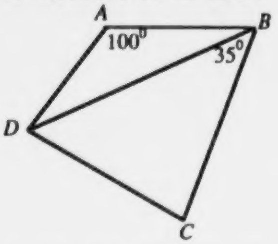
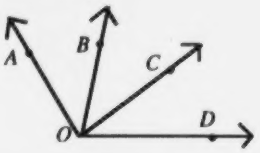
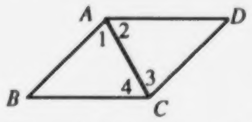
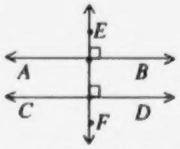
OBJECTIVES	SAMPLE MEASURES
3.2 Use the angle addition postulate to solve problems and write proofs.	3.2.1 Given: *W* is in the interior of $\angle XYZ$, $m\angle XYZ = 113$, and $m\angle XYW = 42$. Find $m\angle WYZ$.  (a) 42 (b) 71 (c) 113 (d) 153 3.2.2 Given: $m\angle 1 = m\angle 3$ and $m\angle MKL = 78$. Find: $m\angle JKN$. Explain how you reached your conclusion.  3.2.3 If $m\angle ABC = 47$ and $m\angle CBD = 18$, find $m\angle ABD$.  (a) 65 (b) 19 (c) 45 (d) 29 3.2.4 If $m\angle ABC = 4x - 5$, $m\angle CBD = 3x$, and $m\angle ABD = 58$, find $m\angle ABC$. 

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will use properties of angles, lines, and planes to solve problems and write proofs.

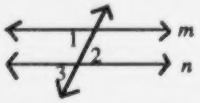
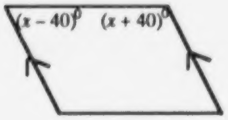
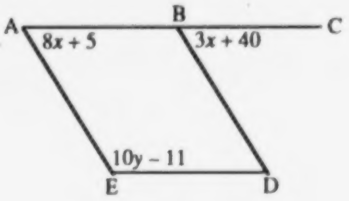
OBJECTIVES		SAMPLE MEASURES	
3.3	Use the definition of angle bisector to solve problems and write proofs.	3.3.1	If $\overrightarrow{BD}$ bisects $\angle ABC$, what is the measure of $\angle ABC$?  (a) 35 (b) 45 (c) 60 (d) 70
		3.3.2	If $\overrightarrow{OB}$ bisects $\angle AOC$, what must be true and why? 
3.4	Use the definitions of parallel lines, perpendicular lines, and perpendicular bisectors to solve problems and write proofs.	3.4.1	In the diagram $\overline{AB} \parallel \overline{CD}$, which pair of angles must be congruent?  (a) $\angle B$ and $\angle D$ (b) $\angle 1$ and $\angle 3$ (c) $\angle 2$ and $\angle 4$ (d) $\angle BAD$ and $\angle BCD$
		3.4.2	What conclusions can be made about $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ in this figure? Justify your answer. Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{EF}$ $\overleftrightarrow{CD} \perp \overleftrightarrow{EF}$ 

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will use properties of angles, lines, and planes to solve problems and write proofs.

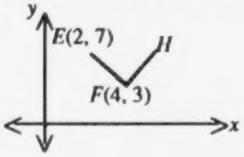
OBJECTIVES	SAMPLE MEASURES
3.5 Use the relationships which exist between special pairs of angles formed by parallel lines and a transversal to solve problems and write proofs.	3.5.1 In the figure, $m \parallel n$. If $m\angle 1 = 2x + 17$ and $m\angle 2 = 5x - 4$, then $m\angle 3 =$ ____?  (a) 31 (b) 7 (c) 149 (d) 139
	3.5.2 Write an equation showing the relationship between the given angles. 
	3.5.3 What values of x and y would enable you to prove that $\overline{AE} \parallel \overline{BD}$ and $\overline{ED} \parallel \overline{AC}$?  A B C $8x + 5$ $3x + 40$ $10y - 11$ E D

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will use properties of angles, lines, and planes to solve problems and write proofs.

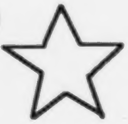
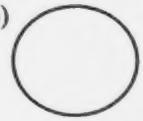
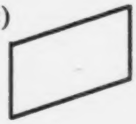
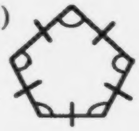
OBJECTIVES	SAMPLE MEASURES
3.6 Use slopes to determine if two lines are parallel or perpendicular.	3.6.1 From this set of linear equations, select a pair whose graphs are parallel lines. (a) $2x + 4y = 5$ (b) $2x - 4y = 5$ (c) $4x + 2y = 5$ (d) $2x + 4y = 7$ 3.6.2 What is the slope of a line perpendicular to the line whose equation is $y = \frac{2x}{3} - 7$? (a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$
3.7 Write the equation of a line parallel or perpendicular to a given line through a given point.	3.7.1 Given the equation of the line $y = 2x + 1$, write an equation of the line parallel to the given line which passes through (0, 3). (a) $y = 2x + 3$ (b) $y = -\frac{1}{2}x + 3$ (c) $y = -\frac{1}{2}x + 1$ (d) $y = 3x$ 3.7.2 Write the equation of a line passing through (0, 10) and perpendicular to $y = 3x - 5$. 3.7.3 If $\angle F$ is a right angle, write the equation of the line which passes through F and H. 

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will use properties of polygons and polyhedra to solve problems and write proofs.

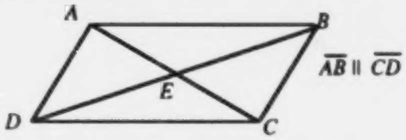
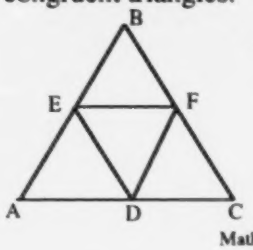
OBJECTIVES		SAMPLE MEASURES	
4.1	Model and describe convex polygons and regular polygons.	4.1.1	Which of the following is a regular polygon? (a)  (b)  (c)  (d) 
		4.1.2	Draw a convex octagon and tell why it is convex.
4.2	Use the measures of the interior and exterior angles of a convex polygon to solve problems.	4.2.1	What is the interior angle sum of a decagon? (a) 360 (b) 1800 (c) 144 (d) 1440
		4.2.2	Explain why the following statement cannot be true: One of the interior angles of a regular polygon has measure 110.
		4.2.3	Using a computer, investigate the relationships of the interior and the exterior angles of a polygon.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will use properties of polygons and polyhedra to solve problems and write proofs.

OBJECTIVES	SAMPLE MEASURES
4.3 Use the properties of proportions to solve problems.	4.3.1 A rope is 36 feet long. It is to be divided into lengths that will be in a ratio of 2:3. What are the two lengths into which the rope will be divided? 4.3.2 If two "similar" rectangles have perimeters of 16 and 36, what is the ratio of the measurement of the two corresponding sides? (a) $\frac{2}{3}$ (b) $\frac{4}{9}$ (c) $\frac{3}{2}$ (d) $\frac{9}{4}$
4.4 Use properties of congruent and similar polygons to solve problems and write proofs.	4.4.1 In the diagram, identify two similar triangles. Use a proof strategy to justify your conclusion.  A B $\overline{AB} \parallel \overline{CD}$ E D C 4.4.2 ΔABC is an isosceles triangle with base $\overline{AC}$. Points D, E, and F are midpoints of $\overline{AC}$, $\overline{AB}$, and $\overline{BC}$, respectively. Identify all congruent triangles. 

MATHEMATICS

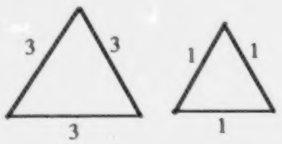
Geometry

GRADE LEVEL: 9-12

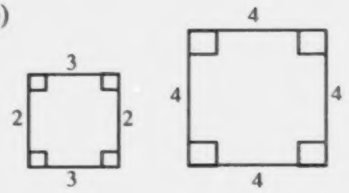
COMPETENCY GOAL 4: The learner will use properties of polygons and polyhedra to solve problems and write proofs.

OBJECTIVES	SAMPLE MEASURES
4.4 Use properties of congruent and similar polygons to solve problems and write proofs.	4.4.3 Which pairs of figures appear to be similar?

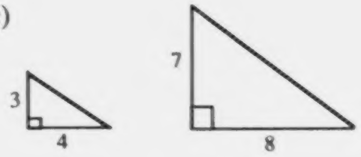
(a)



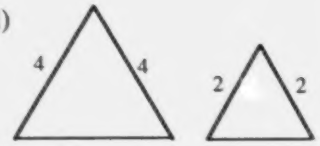
(b)



(c)



(d)

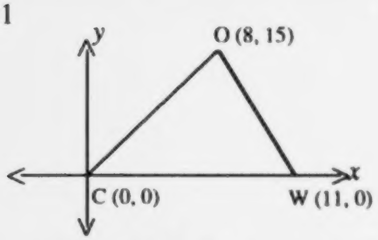


MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will use properties of polygons and polyhedra to solve problems and write proofs.

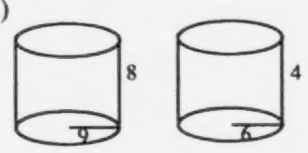
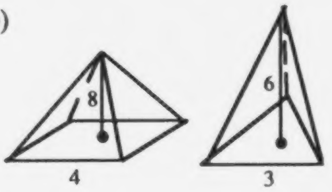
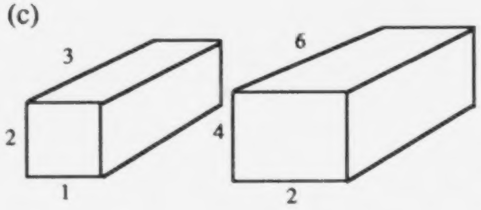
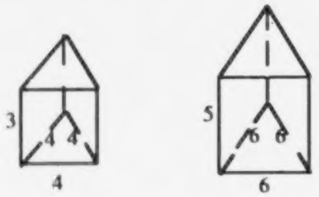
OBJECTIVES	SAMPLE MEASURES
4.5 Investigate a variety of transformations on polygons (tessellation, slide, rotation, flip) in a coordinate plane.	4.5.1  If ΔCOW is slid along the x-axis until C has coordinates of (13, 0), what are the new coordinates of point O? (a) (21, 15) (b) (21, 28) (c) (21, –15) (d) (21, –28)
	4.5.2 Does a regular octagon have 90^{0} rotational symmetry? Justify your answer.
4.6 Model and describe regular and non-regular polyhedra.	4.6.1 Which polyhedron is being described below: " Its base is a regular polygon. Its lateral edges are congruent. All lateral faces are isosceles triangles. The altitude meets the base at its center." (a) Prism (b) Pyramid (c) Right Prism (d) Regular Pyramid
	4.6.2 Build a stick tetrahedron that has four vertices and six edges. Is it possible to build a different one that also has four vertices and six edges but a different number of faces?

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will use properties of polygons and polyhedra to solve problems and write proofs.

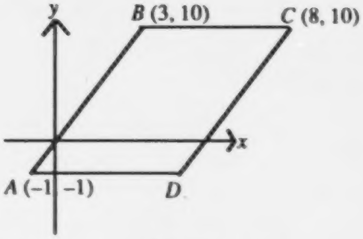
OBJECTIVES	SAMPLE MEASURES
4.7 Identify congruent or similar polyhedra.	4.7.1 Determine which of the following are similar.
	(a) 
	(b) 
	(c) 
	(d) 
	4.7.2 Give four properties of congruent polyhedra.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will use properties of polygons and polyhedra to solve problems and write proofs.

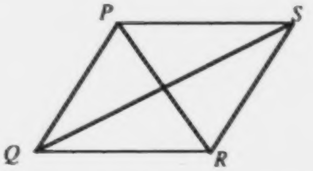
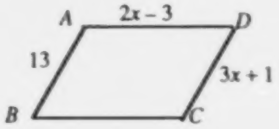
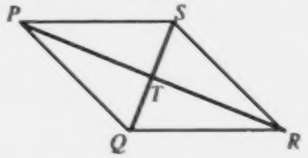
OBJECTIVES	SAMPLE MEASURES
4.8 Determine coordinates of the vertices of polygons in a plane.	4.8.1 In the figure below, if *ABCD* is a parallelogram, what are the coordinates of vertex *D*?  (a) (10, –1) (b) (7, –1) (c) (6, –1) (d) (4, –1)
	4.8.2 Three vertices of a parallelogram are (2, 5), (3, 1), and (–2, –1). Identify the coordinates of a fourth vertex.
4.9 Use coordinate geometry to verify conclusions regarding polygons.	4.9.1 If the coordinates of quadrilateral *ABCD* are *A* (–2, 1), *B* (–3, 4), *C* (9, 8), and *D* (10, 5), what type of quadrilateral is *ABCD*? (a) Trapezoid (b) Rhombus (c) Square (d) Rectangle
	4.9.2 Verify that quadrilateral *ABCD* with vertices *A* (0, 0), *B* (8, 0), *C* (11, 12), and *D* (3, 12) is a parallelogram.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will develop and use properties of quadrilaterals to solve problems and write proofs.

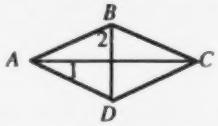
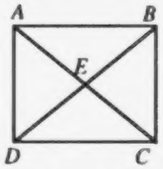
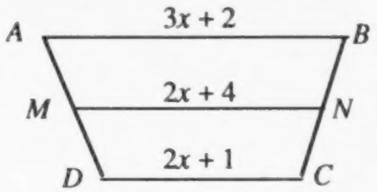
OBJECTIVES	SAMPLE MEASURES
5.1 Use the definition and properties of parallelograms to solve problems and write proofs.	5.1.1 In ▱*PQRS*, $m\angle QPS = 115$ and $m\angle RQS = 25$. Find $m\angle PQS$.  (a) 25 (b) 40 (c) 65 (d) 115 5.1.2 In ▱*ABCD*, find the length of $\overline{BC}$. Justify your answer.  5.1.3 In ▱*PQRS*, $PT = 5x - 8$ and $RT = 3x + 2$. Find *PR*.  (a) 5 (b) 10 (c) 17 (d) 34 5.1.4 In ▱*ABCD*, if two angles are chosen at random, what is the probability that they are congruent? (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will develop and use properties of quadrilaterals to solve problems and write proofs.

OBJECTIVES		SAMPLE MEASURES	
5.2	Develop and use properties of rectangles, rhombi, and squares to solve problems and write proofs.	5.2.1	Quadrilateral *ABCD* is a rhombus. If $m\angle 1 = 35$, them $m\angle 2 =$ ___?  (a) 35 (b) 45 (c) 55 (d) 65
		5.2.2	If *ABCD* is a rectangle, and $AE = 4x$ and $AC = 6x + 5$, find *DB* and justify your answer. 
5.3	Use the definition and properties of trapezoids and isosceles trapezoids to solve problems and write proofs.	5.3.1	In isosceles trapezoid *PQRS*, $\overline{PS} \cong \overline{RQ}$. If $m\angle P = x$, then $m\angle S =$ ___? (a) x (b) $90 - x$ (c) $180 - x$ (d) $2x$
		5.3.2	Given isosceles trapezoid *ABCD* with $\overline{AB} \parallel \overline{CD}$. Prove $\Delta ABC \cong \Delta BAD$ using any proof strategy.
		5.3.3	Given trapezoid *ABCD* with median $\overline{MN}$, find the length of *MN*. 

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 15: The learner will develop and use properties of quadrilaterals to solve problems and write proofs.

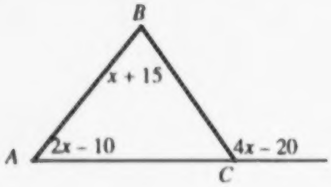
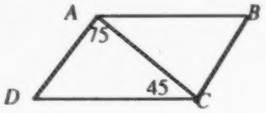
OBJECTIVES	SAMPLE MEASURES
5.4 Given quadrilaterals in a coordinate plane, solve problems and construct proofs.	5.4.1 Find the area of an isosceles trapezoid whose vertices have coordinates (–2, –1), (1, 1), (4, 1), and (7, –1). (a) 24 (b) 12 (c) 2 (d) 9
	5.4.2 Given points *M* (–3, 2), *A* (3, 4), *T* (5, 10), and *H* (–1, 8), show that *MATH* is a rhombus.
	5.4.3 Quadrilateral *PQRS* is a rhombus. Two of the vertices are *P* (3, 2) and *R* (–1, 0). What is the slope of diagonal $\overline{QS}$?

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will develop and use properties of triangles to solve problems and write proofs.

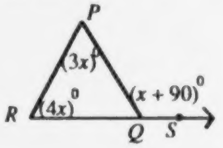
OBJECTIVES	SAMPLE MEASURES
6.1 Classify triangles according to sides and angles.	6.1.1 Which type of triangle is ΔABC?  (a) Scalene (b) Equilateral (c) Isosceles (d) Right 6.1.2 In isosceles ΔABC, an altitude is drawn from vertex A to base $\overline{BC}$ at D. Classify ΔADC according to its sides and angles.
6.2 Explore the relationships of the lengths of the sides of a triangle.	6.2.1 In the diagram below, $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \equiv \overline{AD}$. Which side is the longest side?  (a) $\overline{AD}$ (b) $\overline{CD}$ (c) $\overline{AC}$ (d) $\overline{BC}$ 6.2.2 Using a computer, draw $\overline{AB}$ and $\overline{BC}$ with $AB = 3$ and $BC = 5$. Investigate possible lengths for $\overline{AC}$. State a conjecture about your observations as to maximum and minimum lengths.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will develop and use properties of triangles to solve problems and write proofs.

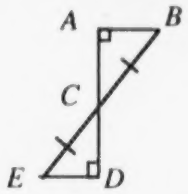
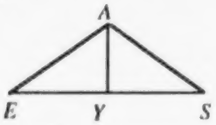
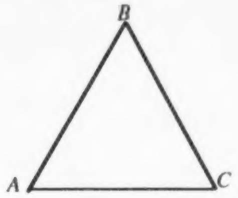
OBJECTIVES		SAMPLE MEASURES	
6.3	Investigate and apply theorems involving the interior and exterior angles of a triangle.	6.3.1	The measures of two angles of a triangle are 40 and 60. Find the measure of the largest exterior angle of the triangle. (a) 80 (b) 120 (c) 140 (d) 100
		6.3.2	Find m ∠P.  (a) 60 (b) 45 (c) 105 (d) 15
		6.3.3	Using a computer, draw different types of triangles. State conjectures about the relationships between the interior angles and the exterior angles.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will develop and use properties of triangles to solve problems and write proofs.

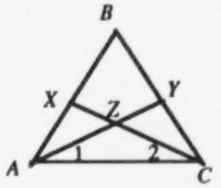
OBJECTIVES		SAMPLE MEASURES	
6.4	Use postulates and theorems to prove that two triangles and their corresponding parts are congruent.	6.4.1	State a congruence method that can be used to prove the triangles congruent. Justify your answer with a proof strategy. 
		6.4.2	If $\angle EAY \cong \angle SAY$, which additional congruent, corresponding parts are needed to prove $\Delta EAY \cong \Delta SAY$ by *SAS*?  (a) $\overline{EY} \cong \overline{SY}$ (b) $\angle E \cong \angle S$ (c) $\overline{EA} \cong \overline{SA}$ (d) $\angle EYA \cong SYA$
6.5	Construct congruent triangles using congruency postulates and theorems.	6.5.1	Construct ΔXYZ congruent to ΔABC by *ASA*. 
		6.5.2	Select a congruency postulate or theorem and explain how you would use it to construct congruent triangles.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will develop and use properties of triangles to solve problems and write proofs.

OBJECTIVES	SAMPLE MEASURES
6.6 Investigate and apply special theorems involving isosceles triangles.	6.6.1 Given isosceles $\triangle ABC$ with base $\overline{AC}$. $\overline{AD}$ bisects $\angle BAC$ and $m\angle B = 44$. Find the measure of $\angle ADB$. (a) 44 (b) 68 (c) 90 (d) 104
	6.6.2 Given: $\overline{AB} \cong \overline{BC}$, $\angle 1 \cong \angle 2$. Is $\overline{AY} \cong \overline{CX}$? Why or why not? 
	6.6.3 If $\triangle ABC$ has coordinates $A\ (0, 0)$, $B\ (10, 0)$, and $C\ (4, 8)$, then which angles, if any, are congruent? (a) $\angle$A and $\angle$B (b) $\angle$B and $\angle$C (c) $\angle$C and $\angle$A (d) None

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will develop and use properties of triangles to solve problems and write proofs.

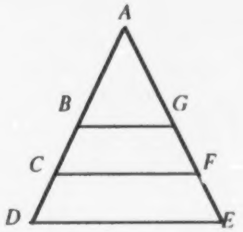
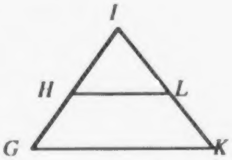
OBJECTIVES		SAMPLE MEASURES	
6.7	Investigate and apply definitions and theorems involving altitudes, perpendicular bisectors, and medians.	6.7.1	If the coordinates of ΔABC are A (–3, –5), B (5, –7), and C (2, 9), then find the length of the median from vertex C.
		6.7.2	The altitudes in a triangle will intersect: (a) in one point on the triangle (b) in one point inside the triangle (c) in one point outside the triangle (d) any one of the above, depending on the type of triangle
		6.7.3	Using a computer, draw as many different types of triangles as possible and investigate the points of intersection of the medians. What conclusions can be reached as to the location of the points of intersection?

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will develop and use properties of triangles to solve problems and write proofs.

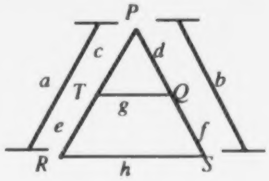
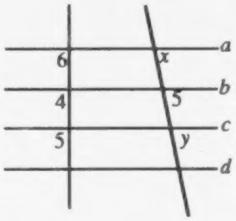
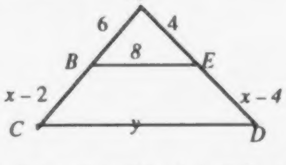
OBJECTIVES	SAMPLE MEASURES
6.8 Investigate and apply the theorem involving the segment joining the midpoints of two sides of a triangle.	6.8.1 Using a computer or construction tools, draw $\triangle ABC$ with D the midpoint of $\overline{AB}$ and E the midpoint of $\overline{BC}$. Draw $\overline{DE}$. What conclusions can be reached as to the relationship of $\overline{DE}$ and $\overline{AC}$? Do this with different types of triangles. Is your conclusion the same for each triangle?
	6.8.2 If B and G are midpoints of $\overline{AD}$ and $\overline{AE}$ and C and F are midpoints of $\overline{BD}$ and $\overline{GE}$ and $BG = 9$, then find the length of $\overline{CF}$.  (a) 4.5 (b) 13.5 (c) 18 (d) 27
	6.8.3 Given: $\overline{GH} \cong \overline{HI}$, $\overline{KL} \cong \overline{LI}$, $HL = 2x - 1$, and $GK = 3x + 3$. Find GK. 

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will develop and use properties of triangles to solve problems and write proofs.

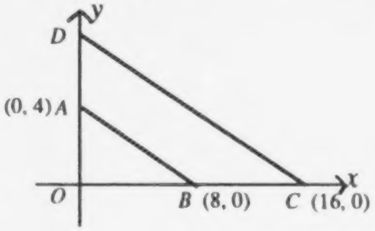
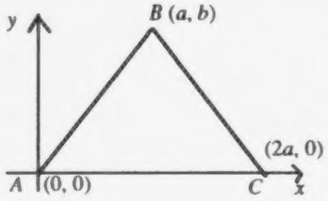
OBJECTIVES	SAMPLE MEASURES
6.9 Apply theorems involving segments divided proportionally.	6.9.1 *If* $\overline{RS} \parallel \overline{TQ}$, name the correct proportion.  P c d a T Q b g e f R h S (a) $\frac{g}{h} = \frac{c}{e}$ (b) $\frac{g}{h} = \frac{a}{c}$ (c) $\frac{g}{h} = \frac{d}{b}$ (d) $\frac{g}{h} = \frac{e}{a}$ 6.9.2 Find the values of x and y if $a \parallel b \parallel c \parallel d$.  6.9.3 If $\overline{BE} \parallel \overline{CD}$, find the perimeter of ΔACD.  (a) 24 (b) 30 (c) 36 (d) 42

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will develop and use properties of triangles to solve problems and write proofs.

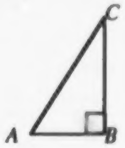
OBJECTIVES	SAMPLE MEASURES
6.10 Given triangles in the coordinate plane, solve problems and write proofs.	6.10.1 If the coordinates of ΔABC are A (4, 8), B (4, –1), and C (–5, –1), find the length of the altitude from B to $\overline{AC}$. (a) $\frac{9}{2}$ (b) $\frac{12}{5}$ (c) $\frac{16}{5}$ (d) $\frac{9\sqrt{2}}{2}$
	6.10.2 Find the coordinates of D if $\overline{AB} \parallel \overline{CD}$. 
	6.10.3 Prove that ΔABC is isosceles. 

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will develop and use properties of right triangles to solve problem.

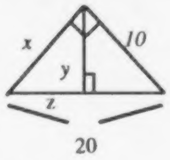
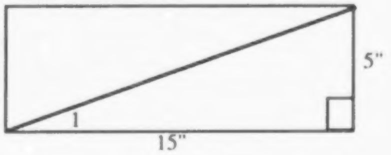
OBJECTIVES	SAMPLE MEASURES
7.1 Find the geometric mean between a pair of numbers.	7.1.1 Find the geometric mean between 4 and 6. (a) $\sqrt{10}$ (b) $4\sqrt{6}$ (c) 5 (d) $2\sqrt{6}$ 7.1.2 Find the geometric mean between 5 and 8.
7.2 Use the Pythagorean Theorem and its converse to solve problems.	7.2.1 Given ΔABC, $AB = 6$, $AC = 9$, find BC.  (a) 45 (b) $3\sqrt{5}$ (c) $5\sqrt{3}$ (d) 117 7.2.2 The dimensions of the graphics screen on the TI-81 are 96 pixels by 64 pixels. What is the length of the diagonal (to the nearest hundredth of a pixel)?
7.3 Use the relationships in a special right triangle to solve problems and write proofs.	7.3.1 The legs of a $45^{0}-45^{0}-90^{0}$ right triangle are five inches long. Find the length of the hypotenuse. (a) $5\sqrt{3}$ (b) 10 (c) $5\sqrt{2}$ (d) $\sqrt{5}$ 7.3.2 Find the length of an altitude of an equilateral triangle if the length of one side is 8 cm.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

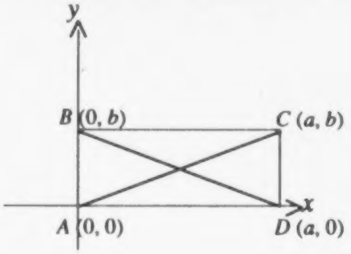
COMPETENCY GOAL 7: The learner will develop and use properties of right triangles to solve problems.

OBJECTIVES		SAMPLE MEASURES	
7.4	State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle.	7.4.1	Find the length of x, y, and z.  (a) $5, 5\sqrt{3}, 15$ (b) $10\sqrt{3}, 5\sqrt{3}, 15$ (c) 15, 5, 8 (d) $10, 5\sqrt{2}, 10$
		7.4.2	Two sides of an open tent form a 90^0 angle at the peak. A support post, placed at a 90^0 angle with the ground, divides the ground line into 2 feet and 8 feet sections. Find the height of the post.
7.5	Use a calculator to apply the definitions of sine, cosine, and tangent to solve problems.	7.5.1	In right triangle ABC, $AC = 6$, $BC = 8$, and $AB = 10$. Find tan B. (a) $\frac{3}{4}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{4}{3}$
		7.5.2	A ladder is leaning against the side of a building. The ladder is 10 meters long and the angle between the ladder and the building is 18^0. How far up the building does the ladder reach (to the nearest hundredth)?
		7.5.3	Two sides of a rectangle are 5" and 15". What is the measure of $\angle 1$? 

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will develop and use properties of right triangles to solve problems.

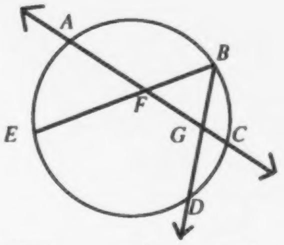
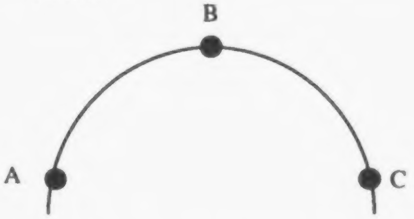
OBJECTIVES	SAMPLE MEASURES
7.6 Use coordinate methods to solve problems and write proofs related to right triangles.	7.6.1 Show that the triangle with vertices *A* (–1, 0), *B* (–5, 8), and *C* (5, 3) is a right triangle. 7.6.2 Justify the following theorem using coordinate geometry: The diagonals of a rectangle are congruent. 

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will develop and use properties of circles and spheres to solve problems and write proofs.

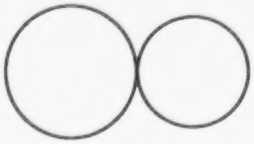
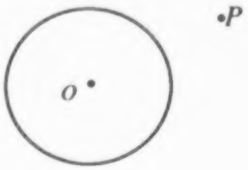
OBJECTIVES	SAMPLE MEASURES
8.1 Identify and use the definition of a circle and sets of points related to the circle.	8.1.1 Which of the following is a chord?  (a) $\overleftrightarrow{AC}$ (b) $\overrightarrow{BD}$ (c) $\overline{EF}$ (d) $\overline{BD}$
	8.1.2 Explain the difference between a secant and a tangent to a circle.
8.2 Find the center of a circle which passes through three non-collinear points.	8.2.1 Explain how to find the center of a circle given three points on a circle. 
	8.2.2 Jennifer found a piece of a wagon wheel. Explain how she can find the diameter of the original wheel.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will develop and use properties of circles and spheres to solve problems and write proofs.

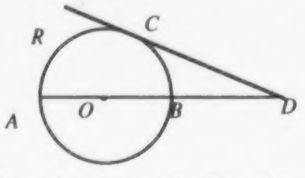
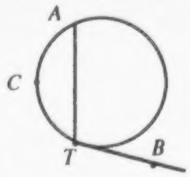
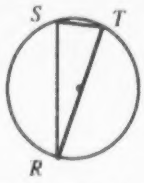
OBJECTIVES		SAMPLE MEASURES	
8.3	Describe the relationship between tangents and circles.	8.3.1	The circles shown have how many common tangents?  (a) 1 (b) 2 (c) 3 (d) 4
		8.3.2	Draw two circles having no common internal tangent and two common external tangents.
		8.3.3	Given a point *P* outside a circle *O*, construct a tangent to that circle through the given point. 

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will develop and use properties of circles and spheres to solve problems and write proofs.

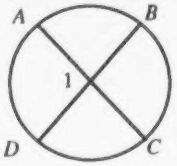
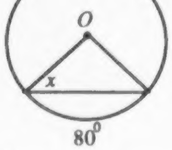
OBJECTIVES		SAMPLE MEASURES	
8.4	Apply the properties involving arcs and angles of circles.	8.4.1	In circle O, $m\overset{\frown}{ARC} = 100$. Find $m\angle D$.  (a) 20 (b) 90 (c) 50 (d) 10
		8.4.2	$\overline{BT}$ is a tangent to the circle. If $m\overset{\frown}{ACT} = 70$, find $m\angle ATB$. 
		8.4.3	$\overline{RT}$ is a diameter of circle O; $m\,\overset{\frown}{RS} = 128$. Find $m\,\angle R$.  (a) 64 (b) 52 (c) 26 (d) 128

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will develop and use properties of circles and spheres to solve problems and write proofs.

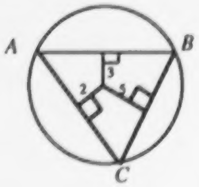
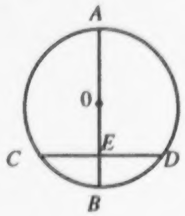
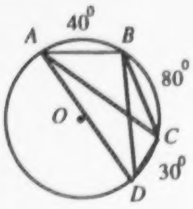
OBJECTIVES	SAMPLE MEASURES
8.4 Apply the properties involving arcs and angles of circles.	8.4.4 Given $m\widehat{AB} = 80$ and $m\widehat{DC} = 70$, find the measure of angle 1.  (a) 75 (b) 80 (c) 105 (d) 150 8.4.5 Given circle O, find x.  (a) 50^{0} (b) 80^{0} (c) 100^{0} (d) 40^{0}

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will develop and use properties of circles and spheres to solve problems and write proofs.

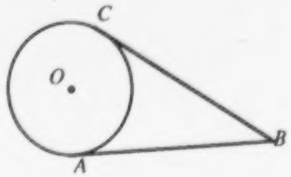
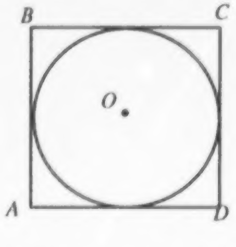
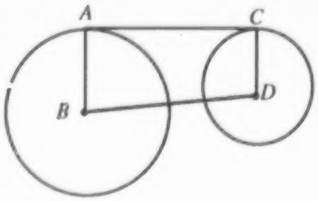
OBJECTIVES	SAMPLE MEASURES
8.5 Apply theorems that relate to the chords of a circle, excluding the product theorem.	8.5.1 Which is the longest chord in the diagram?  (a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{BC}$ 8.5.2 $AB = 16$, $CD = 10$, and $\overline{AB} \perp \overline{CD}$, Find OE.  8.5.3 Explain why there are no congruent chords in circle O. 

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will develop and use properties of circles and spheres to solve problems and write proofs.

OBJECTIVES	SAMPLE MEASURES
8.6 Apply theorems that relate to the tangents, secants, and radii of a circle, excluding the product theorems.	8.6.1 $\overline{BC}$ and $\overline{BA}$ are tangent to circle O. If $BC = 12$, and the radius of circle O is 5, then find OB.  (a) 7 (b) 13 (c) 17 (d) 60 8.6.2 $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$ are tangent to circle O and $AD = 10$, $DC = 15$, and $BC = 14$. Find AB.  8.6.3 The radius of circle B is 6; the radius of circle D is 4. If $AC = 15$, what is BD? 

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will develop and use properties of circles and spheres to solve problems and write proofs.

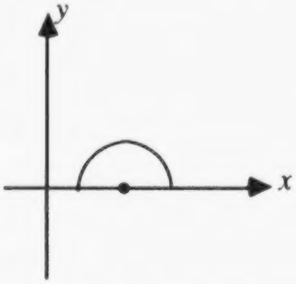
OBJECTIVES	SAMPLE MEASURES
8.7 Describe the relationship between the equation of a circle, its center and radius length.	8.7.1 Write an equation of a circle with center (4, –2) and radius 8. (a) $(x-4)^2+(y-2)^2=64$ (b) $(x-4)^2+(y+2)^2=8$ (c) $(x-4)^2+(y+2)^2=64$ (d) $(x-4)^2+(y-2)^2=8$
	8.7.2 What is the center and radius length of the circle whose equation is: $(x-4)^2+(y+8)^2=100$? (a) center (4, 8); $r=10$ (b) center (–4, 8); $r=10$ (c) center (4, –8); $r=10$ (d) center (4, –8); $r=100$
	8.7.3 Find the equation of the circle with center (5, 7) that is tangent to the *x*-axis.
8.8 Discuss the relationships of congruent, similar, and concentric circles.	8.8.1 Which term best describes an archery target? (a) Congruent circles (b) Similar circles (c) Concentric circles (d) Tangent circles
	8.8.2 What is meant by the terms congruent, similar, and concentric with regard to circles? Give examples of each.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will develop and use properties of circles and spheres to solve problems and write proofs.

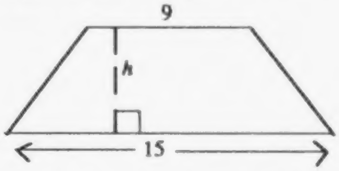
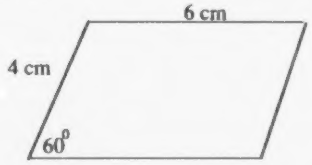
OBJECTIVES		SAMPLE MEASURES	
8.9	Explore spheres and sets of points related to the sphere.	8.9.1	Complete the following analogy: Square: Cube as Circle: ______. (a) Prism (b) Sphere (c) Cylinder (d) Cone
		8.9.2	Explain the result obtained by rotating a semi-circle about the x-axis. 
		8.9.3	Find the area of the circle formed when a plane passes 6 cm from the center of a sphere with radius 10 cm. Round your answer to the nearest hundredth.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will understand and use perimeter, area, and volume formulas to solve problems.

OBJECTIVES	SAMPLE MEASURES
9.1 Find the perimeter of a geometric figure.	9.1.1 Find the perimeter of ΔABC with vertices A (1, 2), B (4, 2), and C (4, –2). (a) 9 (b) 12 (c) 11 (d) 15 9.1.2 If the base of a rectangle is 3 cm and its area is 15 cm^2, what is its perimeter?
9.2 Find the area of a triangle, parallelogram, trapezoid, and rectangle.	9.2.1 The quadrilateral shown below has an area of 400. What is the length of the altitude?  (a) 10 (b) 20 (c) 30 (d) 40 9.2.2 Find the area of the parallelogram below:  (a) 24 cm^2 (b) 12 cm^2 (c) $12\sqrt{3}$ cm^2 (d) $24\sqrt{3}$ cm^2 9.2.3 Find the area of an equilateral triangle whose perimeter is 12 inches. 9.2.4 Find the area of a rectangle whose vertices are A (–1, 2), B (5, 2), C (5, 5) and D (–1, 5).

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will understand and use perimeter, area, and volume formulas to solve problems.

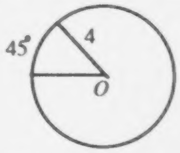
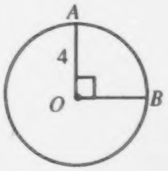
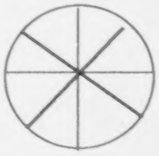
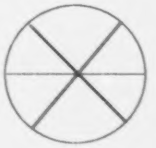
OBJECTIVES	SAMPLE MEASURES
9.3 Find the ratio of the perimeters, areas, and volumes of similar geometric figures.	9.3.1 Two similar pyramids have heights 3 and 12. What is the ratio of their volumes? (a) 1:64 (b) 1:4 (c) 1:16 (d) 1:15 9.3.2 If the lengths of the sides of a triangle are multiplied by 3, what is the ratio of the area of the new triangle to the area of the old triangle? 9.3.3 A square pyramid has a slant height of 5 cm and lateral area 60 cm^2. A similar pyramid has lateral area 240 cm^2. Find the ratio of the volumes of the smaller to the larger pyramid.
9.4 Find the circumference and area of a circle.	9.4.1 The area of a circle is 36π. What is its circumference? (a) 6π (b) 12π (c) 18π (d) 72π 9.4.2 Which choice gives you the most pizza, two 12-inch pizzas or one 17-inch pizza? Explain your answer.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will understand and use perimeter, area, and volume formulas to solve problems.

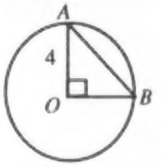
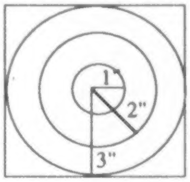
OBJECTIVES	SAMPLE MEASURES
9.5 Compute arc lengths and the area of sectors of a circle.	9.5.1 The area of a sector of the circle with an arc measure of 45^0 and with a radius of 4 is:  (a) 16π (b) 8π (c) 2π (d) 2 9.5.2 What is the length of $\widehat{AB}$?  (a) $4\sqrt{2}$ (b) 2π (c) 4π (d) 16π 9.5.3 If you liked pizza, which would you choose, and why?  A 16-inch pizza to share equally with seven of your friends?  A 14-inch pizza to share equally with five of your friends?

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will understand and use perimeter, area, and volume formulas to solve problems.

OBJECTIVES		SAMPLE MEASURES	
9.6	Use areas to solve problems involving geometric probability.	9.6.1	If a point is chosen at random in the interior of circle *O*, what is the probability that the point is in the interior of ΔAOB?  (a) $\frac{2}{\pi}$ (b) $\frac{1}{4}$ (c) $\frac{5}{2\pi}$ (d) $\frac{1}{2\pi}$
		9.6.2	If a dart is thrown randomly and lands inside the square, what is the probability that it hits the bulls-eye? 
9.7	Find the lateral area, total area, and volume of a right prism, pyramid, right circular cylinder and cone.	9.7.1	Find the volume of a chocolate cake with a 60^0 slice removed if the height of the cake is 6 inches and the diameter is 8 inches. (a) 50.24 in.^3 (b) 251.2 in.^3 (c) 301.44 in.^3 (d) 1205.76 in.^3
		9.7.2	What is the total surface area of a regular square pyramid with base edge 8 and height 6?

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will understand and use perimeter, area, and volume formulas to solve problems.

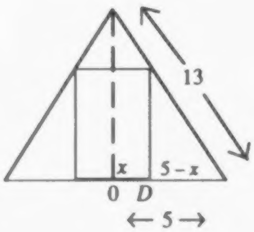
OBJECTIVES		SAMPLE MEASURES	
9.7	Find the lateral area, total area, and volume of a right prism, pyramid, right circular cylinder and cone.	9.7.3	A cubic foot of water is about 7.5 gallons. Calculate how many gallons of water are required to fill a pool that is 30 ft long, 18 ft wide, and whose depth starts at 4 ft and slopes down to an 8 ft depth at the opposite end. (Assume a constant slope.)
		9.7.4	Find the lateral area and volume of a cone whose height is 12 cm and radius is 3 cm.
9.8	Compute the surface area and volume of a sphere to solve problems.	9.8.1	An ice cream cone has a volume of 12π cm^3. A single scoop of ice cream 4 cm in diameter is placed on top. If the ice cream melts into the cone, will it overflow? Justify your answer.
		9.8.2	A fraternity at North Carolina State University wants to paint the "Dean Dome" red. Assume the "Dean Dome" is hemispherical and that the diameter is 200 feet. If one can of paint covers 120 $ft.^2$, how many cans of paint should they buy? Explain your answer.

MATHEMATICS

Geometry

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will understand and use perimeter, area, and volume formulas to solve problems.

OBJECTIVES		SAMPLE MEASURES	
9.9	Use an automatic grapher to solve "Max-Min" problems.	9.9.1	A rectangular enclosure is made with 100 ft of fencing on three sides. The fourth side is the wall of a barn. Find the greatest possible area of such an enclosure. (a) 400 ft^2 (b) 625 ft^2 (c) 1111.1 ft^2 (d) 2500 ft^2
		9.9.2	A rectangle is inscribed in an isosceles triangle with legs 13 and base 10. There are many such rectangles. If $OD = x$, then the area of any such rectangle is: $A = \frac{24x(5-x)}{5}$. Using a graphing calculator, find the value of x for which the greatest area occurs. 

NOTES

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will solve, graph and use equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
1.1 Solve literal equations and formulas for a specified variable.	1.1.1 Solve for y: $a = \frac{y}{r} - 2x$ (a) $y = ra + 2rx$ (b) $y = ra + 2x$ (c) $y = ra - 2rx$ (d) $y = ra - 2x$
	1.1.2 The formula for Newton's Law of Universal Gravity states that the force of gravitational attraction between two bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers: $F = G\frac{Mm}{r^2}$ Solve the formula for r, the distance between the centers of the two masses.
	1.1.3 Solve for x if $r = \frac{a + x}{b + x}$
1.2 Use an automatic grapher to estimate solutions of linear and absolute value equations and inequalities.	1.2.1 Use an automatic grapher to estimate the solution of: $\frac{3}{4}x - 2 = \frac{1}{2}$ (a) {3.33} (b) {1.66} (c) {−3.33} (d) {− 1.66}
	1.2.2 Use an automatic grapher to estimate to the nearest tenth the solution to $\lvert 3x - 1 \rvert = 4$.

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will solve, graph and use equations and inequalities.

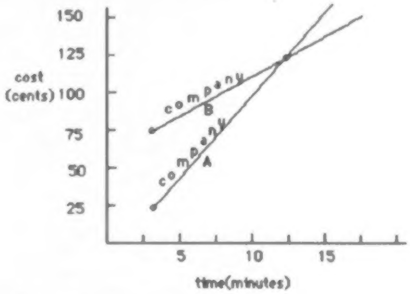
OBJECTIVES	SAMPLE MEASURES
1.3 Solve and graph equations and inequalities involving absolute value.	1.3.1 Solve: $2\lvert x - 3\rvert = 12$ (a) {9} (b) {–3, 9} (c) {–3} (d) {–9, 3}
	1.3.2 Explain why an equation involving absolute value could have two solutions.
1.4 Interpret the slope and intercepts of a line.	1.4.1 The graph of $2x + 3y = 6$ has a slope of: (a) $-\frac{3}{2}$ (b) -2 (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$
	1.4.2 Find the y–intercept of the graph of $2x + 3y = 6$.
	1.4.3 The price of milk in cents is modeled by the linear equation $P = 40Q + 25$, where P is the price of a carton that holds Q quarts of milk. What are the units of the slope of this line? What real-world quantity does the slope represent? What does the price-intercept represent?
1.5 Apply the concepts of parallel and perpendicular lines as determined by a comparison of their slopes.	1.5.1 Any line parallel to the graph of $x - 2y = 4$ has a slope of: (a) -2 (b) 2 (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$
	1.5.2 Write an equation of a line parallel to the graph of $x + y = 3$.
	1.5.3 Find the equation of a line with y–intercept 3 that is parallel to the line $2x + 3y = 6$.
	1.5.4 If the line with equation $ax - 2y = 5$ is perpendicular to the line with equation $x + 3y = 1$, then what is the value of a?

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will solve, graph and use equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
1.6 Write and use an equation of a line which models a set of linear data.	1.6.1 The following graph represents long distance charges for two telephone companies. Which of the companies charges less for a 10-minute telephone call?  (a) Company A (b) Company B (c) same cost (d) can not be determined 1.6.2 Sketch a set of data that is nearly linear and discuss the meaning of the relationship. 1.6.3 What is meant when we say that the relationship between time and distance at a constant rate is linear?

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: The learner will solve, graph and use equations and inequalities.

OBJECTIVES

1.7 Use an automatic grapher to fit a line to a set of linear data. Interpret the slope, intercepts and quality of fit.

SAMPLE MEASURES

1.7.1 The weather reporter tells you that at winds of 10 mph the temperature feels colder than the thermometer reads. This is called wind chill. The following data are reported. Temperatures are all given in degrees Farenheit.

Temperature	35	30	25	20	15	10	5	0
Wind Chill	21	16	9	2	-2	-9	-15	-22

Use your calculator to fit a line to the data and write the equation for this line.

1.7.2 Use your line to predict the wind chill when the temperature is 23^0 and -10^0.

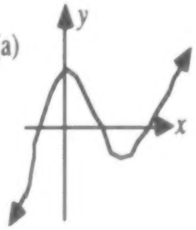
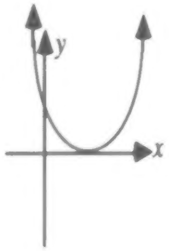
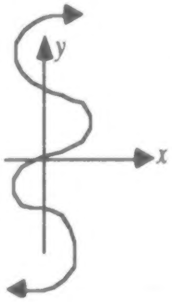
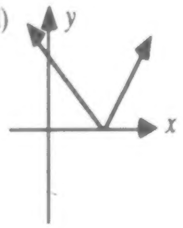
1.7.3 Describe the meaning of the slope in the equation using the words "wind chill" and "temperature".

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of relations and functions.

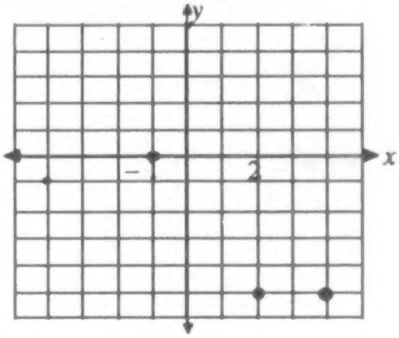
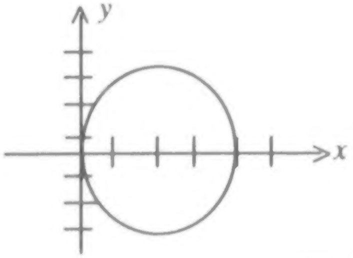
OBJECTIVES	SAMPLE MEASURES
2.1 Determine if a given relation is a function.	2.1.1 A function f includes these ordered pairs: $(-3, 0)$, $(0, 1)$, $(2.5, -1.3)$. f will still be a function when which ordered pair is included in f? (a) $(-3, 2)$ (b) $(0, 0)$ (c) $(1, 0)$ (d) $(-3, -1.3)$ 2.1.2 Sketch a graph that would be reasonable to model the height of a baseball after being hit into the air. Explain in writing whether you think this does or does not represent a function. 2.1.3 Which of the graphs below do not represent functions? (a)  (b)  (c)  (d) 

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of relations and functions.

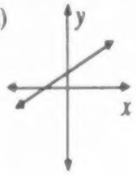
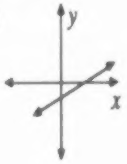
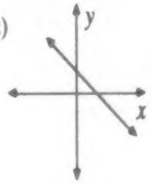
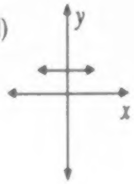
OBJECTIVES	SAMPLE MEASURES
2.2 Identify the domain and range of a relation.	2.2.1 What is the domain for the relation shown in the graph?  (a) all real numbers (b) {–4, –3, –2, –1, 0, 1, 2, 3, 4} (c) {–3, –1, 0, 1, 3} (d) {–4, –1, 0, 2, 4} 2.2.2 You are preparing to construct a graph which shows the relationship between the number of hours worked in a week and the pay earned. What is a reasonable domain for your graph? Justify your answer. 2.2.3 Identify the domain and range of this circle. 

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of relations and functions.

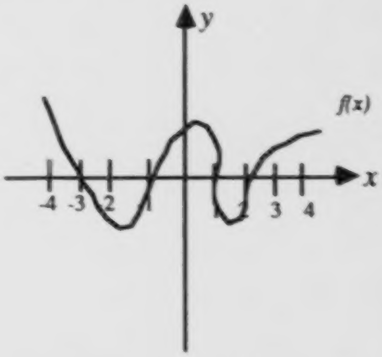
OBJECTIVES	SAMPLE MEASURES
2.3 Use function notation.	2.3.1 If $f(x) = 3x - 5$, find the value of $f(-1)$. (a) -8 (b) $\frac{4}{3}$ (c) 2 (d) -2
	2.3.2 Give an example of a function that had a value of 6 when x was 1.
	2.3.3 If $f(x) = 2^x - x$, then what is the value of $f(3)$?
	2.3.4 If the function f is defined by $f\colon x \rightarrow 2x - \lvert x \rvert$, then f would map -3 onto which number? (a) -9 (b) -3 (c) 3 (d) 9
2.4 Graph relations and functions with and without an automatic grapher.	2.4.1 Choose the graph which most likely fits the equation of $y = mx + b$, $m > 0$, $b > 0$. (a)  (b)  (c)  (d) 
	2.4.2 Graph $x^2 + y^2 = 16$ using an automatic grapher.

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of relations and functions.

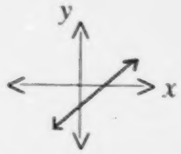
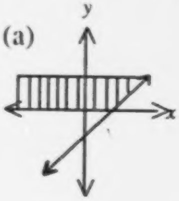
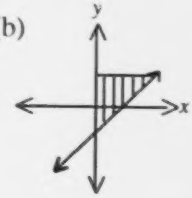
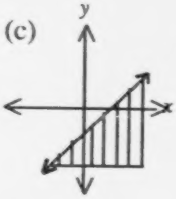
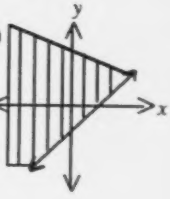
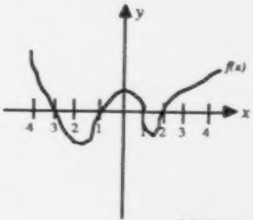
OBJECTIVES	SAMPLE MEASURES
2.5 Find the zeros of a function by examining a graph.	2.5.1 Use an automatic grapher to estimate the zero(s) of the function: $y = x^2 + 3x - 5$ (a) $\{-1.6, -7.24\}$ (b) $\{-4.19, 1.19\}$ (c) $\{0, -2.7\}$ (d) $\{-2.7, 1.10\}$
	2.5.2 What are the values to the nearest integer of the zeros of the function graphed below? 
	2.5.3 Describe how you would use an automatic grapher to find the zeros of a function.
	2.5.4 A polynomial function $f(x)$ has values $f(2) = -5$, $f(5) = 3$, and $f(8) = 10$. Between which to x-values is $f(x)$ sure to have a zero? What x-value would the bisection algorithm lead you to guess for the zero?

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of relations and functions.

OBJECTIVES	SAMPLE MEASURES
2.6. Solve an inequality by examining the graph of the function.	2.6.1 Using an automatic grapher to graph $y = 3x - 2$ yields the graph shown:  Which graph below shows the set of points satisfying $y \geq 3x - 2$? (a)  (b)  (c)  (d)  2.6.2 Given the graph of $y = f(x)$ shown below, what is the solution in terms of x of the inequality $f(x) < 0$? 

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of relations and functions.

OBJECTIVES	SAMPLE MEASURES
2.7 Communicate graphically, algebraically, and verbally real world phenomena as functions.	2.7.1 One-quart cartons of milk sell for \$.65 each, and two-quart cartons sell for \$1.05 each. Assume that the cost of a carton of milk varies linearly with the number of quarts the carton holds. Write a linear equation expressing price as a function of quarts that contains the two points given here.
	2.7.2 The price of a consumer good is \$29.95 if the number of items available to consumers is 100,000. For each 10,000 increase in the number of items, the price drops by \$1.00. Write a function for the total revenue (price per item times number of items).
	2.7.3 Sketch a graph that represents the temperature over time of an ice-cold drink left in a warm room for a period of time.
2.8 Find the composition of functions.	2.8.1 Let $f(x) = x^2 - 2x + 3$ and $g(x) = -2x + 1$. Find $f(g(x))$. (a) $4x^2 + 5$ (b) $4x^2 - 2$ (c) $4x^2 + 2$ (d) $-2x^2 + 4x + 2$
	2.8.2 If $f(g(x)) = 2x^2 + 5$, write a possible expression for $f(x)$ and $g(x)$.

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of relations and functions.

OBJECTIVES	SAMPLE MEASURES

2.9 Use iterative definitions of functions.

2.9.1 In Ourtown, N.C., drivers caught speeding are fined according to this table:

speeder's rate	amount of fine
35 mph	no fine
40 mph	$40
45 mph	40 mph fine + 10% of 40 mph fine
50 mph	45 mph fine + 10% of 45 mph fine

Clocked speeds are always rounded up to the next multiple of 5 so 36 mph would be treated as 40 mph. The fine for each level is determined by taking the fine for the speed 5 mph less and adding 10% of that same amount to it. (rounding to the nearest cent.)

What is the fine in Ourtown for a speeder clocked at 57 mph?

(a) $58.56 (b) $64.42

(c) $70.86 (d) $77.95

2.9.2 If $f(0) = 2$ and $f(x + 1) = 2 \cdot f(x) - 3$, find $f(6)$.

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: The learner will demonstrate an understanding of relations and functions.

OBJECTIVES	SAMPLE MEASURES
2.9 Use iterative definitions of functions.	2.9.3 $f(x)$ is devined recursively by $f(x + 1) = .5f(x)$. If $f(1) = 16$, explain how you could find $f(4)$ and $f(0)$.
	2.9.4 You are trying to pay off your credit card on which you currently owe \$300. The company charges 1.5% interest each month and you pay off \$50 each month. Write an expression for the amount you owe in the (n + 1)th month as a function of the amount you owe in the nth month. (a) $f(n + 1) = 1.5(f(n)) - 50$ (b) $f(n + 1) = f(n) + 1.5f(n) - 50$ (c) $f(n + 1) = f(n) + .015f(n) - 50$

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will operate with matrices.

OBJECTIVES	SAMPLE MEASURES
3.1 Organize data into an array or a matrix.	3.1.1 On a typical day the Yum Yum Ice Cream Shop sells cups of ice cream and frozen yogurt in the following quantities: Ice Cream: Chocolate - 55 Vanilla - 65 Strawberry - 35 Yogurt: Chocolate - 50 Vanilla - 60 Strawberry - 40 The Store Next Door sells in the following quantities: Ice Cream: Chocolate - 60 Vanilla - 70 Strawberry - 50 Yogurt: Chocolate - 80 Vanilla - 80 Strawberry - 55 (a) Represent the number of cups sold at the Yum Yum Ice Cream Shop as a matrix. (b) Represent the number of cups sold at The Store Next Door as a matrix.
	3.1.2 The Widget Manufacturing Company manufactures 2 grades of widgets - the deluxe model and the economy model. The deluxe model requires 2 hours of assembly time, three hours of finishing time and 1/2 hour of packaging time. The economy model requires 1 1/2 hours of assembly time, 2 hours of finishing time and 1/2 hour of packaging time. Store this information in a matrix.

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will operate with matrices.

OBJECTIVES	SAMPLE MEASURES
3.1 Organize data into an array or matrix.	3.1.3 Arrange the following data in a matrix: A school has 3 grades, 10, 11, and 12. In the tenth grade there are 113 boys and 126 girls. In the eleventh grade there are 98 boys and 103 girls. In the twelfth grade there are 89 boys and 95 girls.
3.2 Add and subtract matrices with and without calculators.	3.2.1 Let $A = \begin{bmatrix} 1 & 3 & -2 \\ 4 & 0 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 2 & 1 \\ -3 & 4 & -1 \end{bmatrix}$ Find A + B. (a) $\begin{bmatrix} 6 & 6 & -2 \\ -12 & 0 & -7 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -5 & 1 & -3 \\ 7 & -4 & 8 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & 5 & -1 \\ 1 & 4 & 6 \end{bmatrix}$ 3.2.2 Let $A = \begin{bmatrix} 1 & 4 \\ 3 & 0 \\ -2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 3 & -2 \\ 1 & 0 \end{bmatrix}$. Find A - B. 3.2.3 Matrix A below represents the number of CD's and tapes owned by eleventh grade and twelfth grade students in School A. Matrix B represents the same data for School B. What matrix would reflect the total number of CD's and tapes for both schools? $A = \begin{array}{c} \text{CD's/Tapes} \\ \begin{bmatrix} 12 & 5 \\ 9 & 7 \end{bmatrix} \end{array}$ $B = \begin{array}{c} \text{CD's/Tapes} \\ \begin{bmatrix} 15 & 4 \\ 13 & 6 \end{bmatrix} \end{array}$

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will operate with matrices.

OBJECTIVES	SAMPLE MEASURES
3.3 Use scalar multiplication and multiply matrices with and without calculators.	3.3.1 Perform the indicated operations: $2\begin{bmatrix} 3 & -1 \\ -2 & 0 \end{bmatrix} + 3\begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix}$ (a) $\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 5 \\ -4 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ -4 & -9 \end{bmatrix}$
	3.3.2 Given matrices A and B, such that $A = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 5 \end{bmatrix}$, find AB.
	3.3.3 As a part of an economics project, you and a friend "invested" in three stocks and followed them for a year. You each agreed to buy 1000 shares of stock. You bought 500 shares of stock A, 300 shares of stock B, and 200 shares of stock C. Your friend bought 300 of both A and B and 400 of stock C. You each paid \$12.25 per share for stock A, \$9.75 per share for stock B, and \$30.50 per share for stock C. You both sold stock A for \$15.50, stock B for \$14.75 per share, and stock C for \$29.90 per share. What matrix product would show the amount that you each invested and the amount you had when you sold the stock?

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will operate with matrices.

OBJECTIVES	SAMPLE MEASURES
3.4 Solve real-world problems using matrices.	3.4.1 A toy store's inventory includes small cars (S) in red (r), blue (b), yellow (y), and green and large cars (L) in the same colors. In June the sales could be represented by the matrix, $\begin{array}{cc} & \begin{matrix} r & b & y & g \end{matrix} \\ \begin{matrix} S \\ L \end{matrix} & \begin{bmatrix} 10 & 5 & 15 & 20 \\ 7 & 9 & 8 & 6 \end{bmatrix} \end{array},$ and in July the matrix, $\begin{array}{cc} & \begin{matrix} r & b & y & g \end{matrix} \\ \begin{matrix} S \\ L \end{matrix} & \begin{bmatrix} 8 & 10 & 6 & 5 \\ 10 & 5 & 15 & 2 \end{bmatrix} \end{array}.$ Which matrix represents the number of cars sold in the two-month period? (a) $\begin{array}{cc} & \begin{matrix} r & b & y & g \end{matrix} \\ \begin{matrix} S \\ L \end{matrix} & \begin{bmatrix} 17 & 14 & 23 & 26 \\ 18 & 15 & 21 & 7 \end{bmatrix} \end{array}$ (b) $\begin{array}{cc} & \begin{matrix} r & b & y & g \end{matrix} \\ \begin{matrix} S \\ L \end{matrix} & \begin{bmatrix} 18 & 15 & 21 & 25 \\ 17 & 14 & 23 & 8 \end{bmatrix} \end{array}$ (c) $\begin{array}{cc} & \begin{matrix} r & b & y & g \end{matrix} \\ \begin{matrix} S \\ L \end{matrix} & \begin{bmatrix} 10 & 5 & 15 & 20 \\ 10 & 5 & 15 & 2 \end{bmatrix} \end{array}$ (d) $\begin{array}{cc} & \begin{matrix} r & b & y & g \end{matrix} \\ \begin{matrix} S \\ L \end{matrix} & \begin{bmatrix} 8 & 10 & 6 & 15 \\ 7 & 9 & 8 & 6 \end{bmatrix} \end{array}$

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will operate with matrices.

OBJECTIVES	SAMPLE MEASURES
3.4 Solve real-world problems using matrices.	3.4.2 The student store decided to sell school pennants. The art teacher laid out a design on a grid. The vertices of the pennant ended on (6, 6), (6, 16) and (24, 11). To find the area of a triangle in a coordinate system the determinant of a 3 x 3 matrix can be used with the following formula: $\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ The sign is chosen so that a positive area results. What is the area of material needed to make a school pennant? (a) 50 (b) 90 (c) 120 (d) 180
3.5 Write and solve systems of linear equations in matrix form.	3.5.1 Write the matrix equation that you would use to solve the system of linear equations below: $3x + 5y = 19$ and $4x - 7y = 11$. 3.5.2 Solve the system above. 3.5.3 What is the advantage of using matrices for solving a system of equations?

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: The learner will operate with matrices.

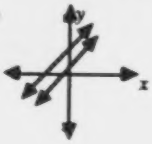
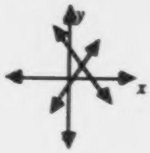
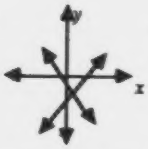
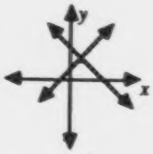
OBJECTIVES	SAMPLE MEASURES
3.6 Solve matrix equations of the form $AX = B$ using calculators or computers.	3.6.1 Use your calculator to solve the matrix equation $AX = B$, where $A = \begin{bmatrix} 2 & -1 & 3 \\ 2 & 0 & -3 \\ 0 & 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ 11 \\ -5 \end{bmatrix}$
	3.6.2 Use your calculator to solve the matrix equation $AX = B$, where $A = \begin{bmatrix} 3 & 5 & -9 \\ 4 & 7 & 2 \\ 6 & -9 & -8 \end{bmatrix}$ and $B = \begin{bmatrix} 26 \\ -7 \\ 3 \end{bmatrix}$
	3.6.3 Use your calculator to solve: $3x + 2y + z = 8$ $2x - y + 2z = -7$ $5x + 3y + 4z = 3$

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will graph and solve systems of equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
4.1 Solve systems of two equations graphically with and without an automatic grapher.	4.1.1 Which graph illustrates the solution to: $x - y = 1$ and $x + y = 5$? (a)  (b)  (c)  (d) 
	4.1.2 Solve graphically: $2x + y = 1$ $-3x + y = 1$
4.2 Solve systems of two equations in two variables.	4.2.1 Solve the following system of equations by using the linear combination method. $2x + y = 6$ $5x - 2y = 4$ (a) $(\frac{8}{9}, \frac{38}{9})$ (b) $(\frac{10}{9}, \frac{22}{9})$ (c) $(\frac{16}{9}, \frac{22}{9})$ (d) $\varnothing$
	4.2.2 Solve the following system of equations using the substitution method: $2x - y = -7$ $6x + 3y = 9$
	4.2.3 Explain the difference between solving by the linear combination method and the substitution method.
	4.2.4 Solve the system of equations: $y = 2x - 3$ $y = x^2 + x - 5.$

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will graph and solve systems of equations and inequalities.

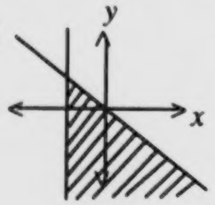
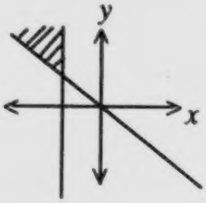
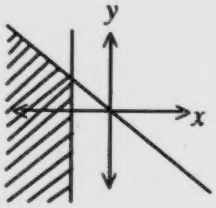
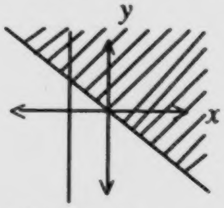
OBJECTIVES	SAMPLE MEASURES
4.3 Solve systems of three equations in three variables.	4.3.1 Solve: $x - 2y + 3z = 9$ $3x - y + z = -7$ $5y + 4z = 2$ (a) (4, 2, 3) (b) (0, 6, –7) (c) (–4, –2, 3) (d) (1, 2, –2) 4.3.2 Solve for x: $x + y - 3z = 8$ $2x - 3y + z = -6$ $3x + 4y - 2z = 20$
4.4 Use systems of equations and inequalities to solve problems.	4.4.1 A company assembly line can produce a widget in 15 minutes or a zumul in 20 minutes. If the company plans to make 4 widgets for every zumul, find the maximum number of widgets that can be produced in an 8-hour shift. (a) 24 (b) 16 (c) 6 (d) 4 4.4.2 State two equations in two variables which would be appropriate to solving this problem: With a tailwind, a plane takes 5 hours to fly from Apex to Cannon, a distance of 800 miles. A plane traveling from Cannon to Apex, against the same wind, takes 8 hours. What is the speed of the wind?

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will graph and solve systems of equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
4.5 Solve systems of inequalities by graphing.	4.5.1 Which graph shows the solution for the system of inequalities $x + y \ge 0$ and $x \le -1$? (a)  (b)  (c)  (d)  4.5.2 Solve by graphing: $-x + y \ge 4.$ $3x - y \le 6.$

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: The learner will graph and solve systems of equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
4.6 Use linear programming to solve real-world problems.	4.6.1 A paper mill produces two grades of paper. Their total output per day is 280 lbs. of paper. The top quality paper sells for \$1.50 per pound and the general quality paper sells for \$1.10 per pound. The production of top quality paper leaves a residue of 0.02 oz. of pollutant #1 and 0.06 oz. of pollutant #2. The production of general quality paper leaves a residue of 0.05 oz. of pollutant #1 and 0.01 oz. of pollutant #2. The federal restriction for pollutants is 120 oz. of pollutant #1 and 136 oz. of pollutant #2 per day. How many pounds of each grade should be produced to maximize the profit for the company? (a) 2,000 lbs. of top grade 1,600 lbs. of general grade (b) 1,600 lbs. of top grade 2,000 lbs. of general grade (c) 2,600 lbs. of top grade 1,000 lbs. of general grade (d) 1,000 lbs. of top grade 2,600 lbs. of general grade
	4.6.2 Why do the vertices of a polygonal region produce the maximum/ minimum values in a linear programming situation?

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will perform operations and solve problems with polynomials.

OBJECTIVES		SAMPLE MEASURES
5.1	Divide one polynomial by another of a lower degree.	5.1.1 Divide: $6a^3 + 5a^2 + 9$ by $2a + 3$. (a) $3a^3 + 5a^2 + 3$ (b) $3a^2 + 2a + 3$ (c) $3a^2 - 2a + 3$ (d) $3a^2 + 7a + 3$ 5.1.2 Divide: $m^3 + 3m^2 - 7m - 21$ by $m^2 - 7$.
5.2	Use synthetic division to divide a polynomial by a linear binomial.	5.2.1 Divide using synthetic division. $\frac{x^3 + 2x^2 - 4}{x - 3}$ (a) $x^2 + 8x + 24 + \frac{68}{(x-3)}$ (b) $x^2 + 5x + 15 + \frac{41}{x-3}$ (c) $x^2 - x + 3 - \frac{14}{(x-3)}$ (d) $x^2 - x - \frac{1}{(x-3)}$ 5.2.2 Show the set-up if you were using synthetic division to divide: $\frac{5x^4 + 2x^3 + 6x - 1}{x + 2}$ 5.2.3 Divide: $x^3 - 2x^2 + 2x - 1$ by $x - 1$.

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will perform operations and solve problems with polynomials.

OBJECTIVES	SAMPLE MEASURES
5.3 Factor polynomials completely.	5.3.1 Factor $12x^2 - 8x - 15$ completely. (a) $(4x + 3)(3x - 5)$ (b) $(6x - 5)(2x + 3)$ (c) $(6x + 5)(2x - 3)$ (d) can't be factored
	5.3.2 Factor: $a^3 + 27$
	5.3.3 Factor completely: $4x^3 + 8x^2 - 12x$.
5.4 Use factoring to solve polynomial equations.	5.4.1 Solve $4x^3 - 10x^2 - 24x = 0$ by factoring. (a) $\{-\frac{3}{2}, 2, 4\}$ (b) $\{-4, \frac{3}{2}, 2\}$ (c) $\{-4, 0, \frac{3}{2}\}$ (d) $\{-\frac{3}{2}, 0, 4\}$
	5.4.2 Solve $x^3 - 9x = 0$ by factoring.

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will perform operations and solve problems with polynomials.

OBJECTIVES	SAMPLE MEASURES
5.5 Use polynomial equations to solve problems.	5.5.1 The perimeter of a rectangle is 44cm and its area is 120cm^2. Find the dimensions of the rectangle. (a) 8 by 15 (b) 14 by 8 (c) 6 by 20 (d) 10 by 12
	5.5.2 An amusement park estimates that for every dollar they lower the price of their admission tickets, an average of 1000 more people per day will visit the park. If they now have an average of 15,000 people per day and charge $15.00 per ticket, what price ticket would insure them of the greatest income?
	5.5.3 The area of a rectangle is 88cm^2 and its length is 5 more than its width. Find the dimensions of the rectangle.
5.6 Expand powers of binomials using Pascal's triangle or the binomial theorem.	5.6.1 Expand: $(a+1)^4$ (a) a^4+1 (b) a^4+4 (c) a^4+4a^2+1 (d) $a^4+4a^3+6a^2+4a^1+1$
	5.6.2 Expand: $(x-3)^5$
	5.6.3 Expand: $(x-2y)^4$

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: The learner will perform operations and solve problems with polynomials.

OBJECTIVES	SAMPLE MEASURES
5.7 Write a polynomial equation given its solutions.	5.7.1 Which of the following equations would have roots –2, 0, 3? (a) $(x+2)(x-3)=0$ (b) $x(x-3)(x+3)=0$ (c) $x(x+2)(x-3)=0$ (d) $x(x-2)(x+3)=0$
	5.7.2 Write an equation for a polynomial that has the solution set $\{-1, 3, 4\}$.
	5.7.3 Write an equation for a polynomial that has roots –2, 3, –4. (a) $x^3+5x^2-10x+24=0$ (b) $x^3-5x^2+10x-24=0$ (c) $x^3+3x^2-10x-24=0$ (d) $x^3-3x^2-10x+24=0$

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will use rational expressions to solve problems.

OBJECTIVES	SAMPLE MEASURES
6.1 Simplify ratios involving algebraic expressions.	6.1.1 Write as a fraction in lowest terms. $\frac{x^2-9}{x^2+x-6}$ (a) $\frac{-3}{x-2}$ (b) $\frac{3}{2}$ (c) $\frac{x+3}{x+2}$ (d) $\frac{x-3}{x-2}$ 6.1.2 Simplify: $\frac{s^2-s}{s^2+2s-2}$
6.2 Use expressions involving negative exponents.	6.2.1 Simplify: $(2x^3y^{-2})^2(xy^2)^{-3}$ (a) $4x^3y^{10}$ (b) $\frac{4}{x^3y^{10}}$ (c) $\frac{4x}{x^3y^{10}}$ (d) $\frac{4x^3}{y^{10}}$ 6.2.2 Write $2xy^{-2}$ without using negative exponents.
6.3 Find sums and differences of rational algebraic expressions.	6.3.1 Simplify the difference: $\frac{5x}{x^2-25}-\frac{2}{x-5}$ 6.3.2 Simplify the sum: $\frac{2}{x^2y}+\frac{5}{x^7y^2}$

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will use rational expressions to solve problems.

OBJECTIVES	SAMPLE MEASURES
6.4 Find products and quotients of rational algebraic expressions.	6.4.1 Multiply: $\frac{4a}{5b} \cdot \frac{15b}{16a}$ (a) $\frac{3}{4}$ (b) $\frac{3a}{b}$ (c) $\frac{3a}{4b}$ (d) $\frac{60a}{80b}$ 6.4.2 Simplify: $\frac{x^2+3x}{x^2+2x-3} \cdot \frac{x^2-x}{x^2-x-2}$. (a) $\frac{x^2}{x-2}$ (b) $x-2$ (c) $\frac{1}{x-2}$ (d) $\frac{x}{x-2}$ 6.4.3 Divide: $\frac{4x^2y}{15a^3b^3} \div \frac{2xy^2}{5ab^3}$ 6.4.4 Simplify: $\frac{x^2-9}{x+2} \div \frac{x+3}{x^2-4}$. (a) $\frac{x-3}{x-2}$ (b) $\frac{x-2}{x-3}$ (c) $(x-3)(x-2)$ (d) $(x-3)^2(x-2)^2$

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will use rational expressions to solve problems.

OBJECTIVES	SAMPLE MEASURES
6.5 Simplify complex fractions.	6.5.1 Simplify: $\dfrac{\frac{1}{5a^2} - \frac{2}{b}}{\frac{7}{10a} + \frac{3}{2b^2}}$ (a) $\dfrac{7ab^2 + 15a^2}{2b^2 - 20a^2b}$ (b) $\dfrac{2b^2 - 20a^2b}{10a^2b^2}$ (c) $\dfrac{2b^2 - 20a^2b}{7ab^2 + 15a^2}$ (d) $\dfrac{7ab^2 + 15a^2}{10a^2b^2}$ 6.5.2 Simplify the fraction: $\dfrac{\frac{x^2}{x^2 - 25y^2}}{\frac{x}{5y - x}}$ 6.5.3 Simplify: $\dfrac{\frac{6}{a+3} + \frac{4}{a-4}}{\frac{2}{a-4} + \frac{5}{a+3}}$

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will use rational expressions to solve problems.

OBJECTIVES	SAMPLE MEASURES
6.6 Solve fractional equations.	6.6.1 Solve: $\frac{5}{n+2} = \frac{3}{2n}$ (a) $\{\frac{6}{7}\}$ (b) $\{\frac{2}{7}\}$ (c) $\{\frac{2}{13}\}$ (d) $\{\frac{7}{6}\}$
	6.6.2 Solve for x: $\frac{2x-9}{x-7} + \frac{x}{2} = \frac{5}{x-7}$
	6.6.3 Solve: $\frac{2}{x+1} + \frac{x-6}{1-x^2} = \frac{3}{x^2-1}$
6.7 Solve problems involving fractional equations.	6.7.1 Paul can plant his wheat crop in 10 days. His daughter can do it in 15 days. How many days will it take if they work together? (a) 7.5 (b) 7 (c) 6 (d) 8
	6.7.2 The ratio of four less than a number to 26 more than a number is one to three. What is the number?

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: The learner will use rational expressions to solve problems.

OBJECTIVES	SAMPLE MEASURES
6.8 Solve problems of direct and inverse variation.	6.8.1 A map is scaled so that 1 cm represents 15 km. How far apart are two towns if they are 7.9 cm apart on the map? (a) 22.9 km (b) 118.5 km (c) 7.1 km (d) 11.9 km
	6.8.2 In a given proportion, n is inversely proportional to p. When $n = 1/27$, $p = 3$. What is n when $p = 1/3$?
6.9 Use joint and combined variation to solve problems.	6.9.1 The heat loss of a glass window varies jointly as the area of the window and the difference between outside and inside temperature. A window that measures 3 feet by 5 feet loses 500 BTU/h when the temperature outside is 10 degrees less than the inside temperature. What is the heat loss through the same window if the difference between outside and inside temperature is 25 degrees? (a) 1000 BTU/h (b) 1250 BTU/h (c) 1500 BTU/h (d) 2000 BTU/h
	6.9.2 How does joint variation differ from direct and inverse variation?

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will solve problems with quadratic equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
7.1 Complete the square to solve quadratic equations.	7.1.1 What number should be added to both members of $x^2 - 7x = -4$ if it is being solved by completing the square? (a) $\frac{7}{2}$ (b) 49 (c) $\frac{49}{4}$ (d) $\frac{49}{2}$
	7.1.2 Solve $x^2 + 8x - 6 = 0$ by completing the square.
7.2 Use the quadratic formula to solve quadratic equations.	7.2.1 Which answer shows how to solve the given equation using the quadratic formula? $2x^2 - 3x - 4 = 0$ (a) $\frac{3 \pm \sqrt{2^2 - 4(2)(-4)}}{2(2)}$ (b) $\frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)}$ (c) $\frac{4 \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2}$ (d) $\frac{3 \pm \sqrt{3^2 - 4(-3)(-4)}}{2(2)}$
	7.2.2 Solve the equation by using the quadratic formula. $3x^2 - 2x - 5 = 0.$

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will solve problems with quadratic equations and inequalities.

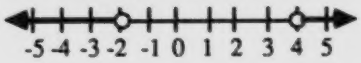
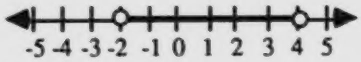
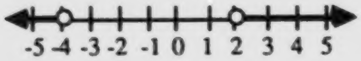
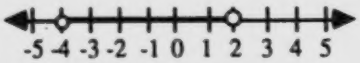
OBJECTIVES	SAMPLE MEASURES
7.3 Define complex numbers and perform basic operations with them.	7.3.1 Which of the following complex numbers is equivalent to $\sqrt{-32}$? (a) $-4i$ (b) $16i$ (c) $4i\sqrt{2}$ (d) $-4i\sqrt{2}$
	7.3.2 Graph the complex number $2 + 3i$.
	7.3.3 Add the complex numbers $2 + 5i$ and $3 - 8i$.
	7.3.4 Find the product of the complex numbers $3 - 2i$ and $2 + 7i$.
	7.3.5 Give an example showing that the product of two complex numbers may not be a complex number.

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will solve problems with quadratic equations and inequalities.

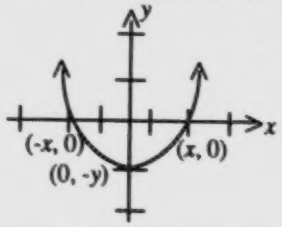
OBJECTIVES	SAMPLE MEASURES
7.4 Solve quadratic inequalities.	7.4.1 Which of the following is a graph of the solution set for the inequality: $x^2 > 2(x + 4)$? (a)  (b)  (c)  (d)  7.4.2 Explain how factoring can help determine the solution set of a quadratic inequality. 7.4.3 Solve for x: $x^2 - 3x < 18$.
7.5 Determine the solutions of quadratic and other polynomial equations using an automatic grapher.	7.5.1 Use an automatic grapher to determine the solutions to this quadratic equation to the nearest hundredth: $x^2 - 4x - 2 = 0$ (a) {–.45, 4.45} (b) {.25, 4.72} (c) {–4.45, 6.25} (d) {–.35, 4.25} 7.5.2 What procedure would you use with an automatic grapher to determine the roots of a quadratic equation?

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will solve problems with quadratic equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
7.6 Solve problems using quadratic equations and inequalities.	7.6.1 Select the quadratic equation you would use to solve the following problem: Find two consecutive odd integers so that the sum of their squares is 290. (a) $x^2 + y^2 = 290$ (b) $x^2 + (x + 1)^2 = 290$ (c) $x^2 + (x + 2)^2 = 290$ (d) $x^2 + (x - 1)^2 = 290$
	7.6.2 The graph of a quadratic function $y = ax^2 + bx + c$ is shown below.  1. What are the solutions of the equation $ax^2 + bx + c = 0$? 2. What are the solutions of the inequality $ax^2 + bx + c < 0$? 3. What are the solutions of the inequality $ax^2 + bx + c > 0$?
	7.6.3 The length of Roanoke Park is six feet more than its width. A walkway three feet wide surrounds the outside of the park. The total area of the walkway is 288 square feet. Find the dimensions of the park.

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will solve problems with quadratic equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
7.7 Interpret the maximum and minimum values of a quadratic function.	7.7.1 The height of a ball is given by $h = -4.9t^2 + 10t + 1$ where t is measured in seconds. How high is the ball in one second? What is the maximum height of the ball? At what time does it hit the ground?
	7.7.2 How do you determine if a quadratic function contains a minimum or a maximum?
	7.7.3 A student has learned that his grade, G, in math is determined by a quadratic function of the number of hours he studies per week, t. The function is $G = -(t - 5)^2 + 92.$ How many hours must he study to achieve his maximum grade? What is his maximum grade? What grade will he make if he does not study?
7.8 Use the discriminant of a quadratic equation to determine the nature of the roots and the number of x-intercepts of the graph.	7.8.1 Use the discriminant of the quadratic equation $2x^2 + 3x = 5$ to determine the nature of the roots and the number of x-intercepts of the graph. (a) Two real rational numbers (b) One real rational number (c) Two imaginary roots (d) One imaginary root
	7.8.2 Compute the discriminant of $5 - 3x - x^2 = 0$. How can this information assist one in knowing the number of x–intercepts of $y = 5 - 3x - x^2$?

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will solve problems with quadratic equations and inequalities.

OBJECTIVES	SAMPLE MEASURES
7.9 Explore the relationship between coefficients and solutions of a quadratic equation.	7.9.1 Write a quadratic equation which has the given solution set {2, – 1/2}. (a) $2x^2+3x-2=0$ (b) $2x^2-3x+2=0$ (c) $2x^2-3x-2=0$ (d) $2x^2+3x+2=0$
	7.9.2 Paul said that in order to find the quadratic equation with solution set {2, – 3}, the first step would be $(x+2)(x-3)=0$. Explain his error.
	7.9.3 Which is the sum of the roots of: $nx^2+cx+k=0$? (a) $\frac{-c}{n}$ (b) $\frac{c}{n}$ (c) $\frac{-k}{n}$ (d) $\frac{k}{n}$
	7.9.4 What conclusion can you state concerning the sum and the product of the solution of: $3x^2-\sqrt{2}\,x+7=0$?
7.10 Solve equations which contain radical expressions.	7.10.1 Solve: $\sqrt{x+9}=9-\sqrt{x}$ (a) {4} (b) {4, –4} (c) {2} (d) {16}
	7.10.2 Explain why {25, 36} is not the solution set for $x+\sqrt{x}-30=0$.

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: The learner will solve problems with quadratic equations and inequalities.

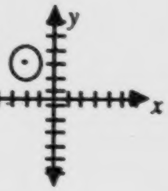
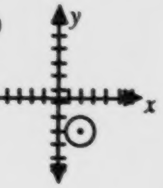
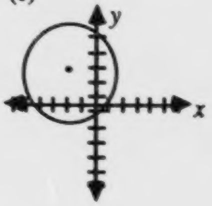
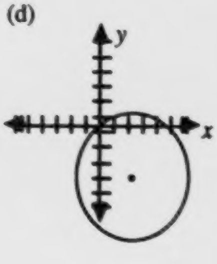
OBJECTIVES	SAMPLE MEASURES
7.11 Explore complex numbers as solutions to quadratic equations.	7.11.1 Find a quadratic equation that has these roots: 5 + 2i 5 − 2i (a) $x^2 + 25x - 4 = 0$ (b) $x^2 - 10x - 29 = 0$ (c) $x^2 + 10x - 29 = 0$ (d) $x^2 - 10x + 29 = 0$
	7.11.2 Solve the quadratic equation over the complex numbers: $2x^2 - x + 7 = 0.$

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will use analytic geometry to solve problems.

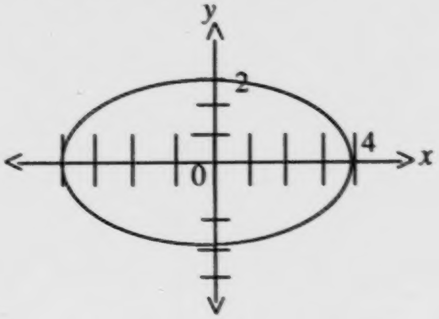
OBJECTIVES	SAMPLE MEASURES
8.1 Use the distance and midpoint formulas.	8.1.1 The distance between the points having coordinates (2, 3) and (1, 4) is: (a) 10 (b) $\sqrt{2}$ (c) $\sqrt{10}$ (d) $5\sqrt{2}$
	8.1.2 The endpoints of the diameter of a circle are (4, –2) and (–12, 4). Find the center of the circle.
	8.1.3 The three vertices of a triangle are (2, 3), (–5, 4) and (–5, 3). Find the perimeter of the triangle.
8.2 Write the equations of and graph circles and parabolas given their geometric properties.	8.2.1 Which of the following is the graph of $(x-2)^2 + (y+3)^2 = 4$? (a)  (b)  (c)  (d) 

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 8: The learner will use analytic geometry to solve problems.

OBJECTIVES		SAMPLE MEASURES	
8.2	Write the equations of and graph circles and parabolas given their geometric properties.	8.2.2	Write an equation for a circle with center (5, 2) and radius 4.
		8.2.3	Given the equation of a circle, $x^2 + 4x + y^2 - 6y = -9$, find the equation of a line tangent to the circle at the point (0, 3).
8.3	Explore the equations of and graph ellipses and hyperbolas.	8.3.1	The graph shown would be the graph of which equation?  (a) $\frac{x^2}{4} - \frac{y^2}{2} = 1$ (b) $\frac{x^2}{4} + \frac{y^2}{2.} = 1$ (c) $\frac{x^2}{16} + \frac{y^2}{4} = 1$ (d) $\frac{x^2}{4} + \frac{y^2}{16} = 1$
		8.3.2	Graph $xy = 12$.

revised, 1992

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will solve problems involving logarithmic and exponential functions.

OBJECTIVES		SAMPLE MEASURES	
9.1	Use expressions involving fractional exponents.	9.1.1	Write $(\sqrt[3]{x})^2$ in exponential form.
		9.1.2	Write $(4x^3)^{\frac{5}{2}}$ in the form ax^c where a and c are real numbers.
9.2	Write an exponential function of the form $f(x) = a \cdot b^x$ given the base and a point.	9.2.1	An exponential function of the form $y = a \cdot 2^x$ passes through the point (3, 4). Find the value of a. (a) 2 (b) $\frac{1}{2}$ (c) 3 (d) $\frac{3}{4}$
		9.2.2	Mary invests $200 at 6% compounded annually. Write an equation of the form $f(x) = a \cdot b^x$ to represent this situation. Use your equation to determine how much money Mary will have after 15 years.
9.3	Graph exponential functions of the form $f(x) = a \cdot b^x$.	9.3.1	Using a calculator, approximate to the nearest tenth the value of x when $y = 2$ in the following expression: $y = 1.2^x$ (a) 3.2 (b) 3.4 (c) 3.6 (d) 3.8
		9.3.2	Graph the function $y = 1.1 \cdot 2^x$ using an automatic grapher.
9.4	Use exponential equations of the form $f(x) = (1 + r)^x$ where r is given as a rate of growth.	9.4.1	$500 is invested in an account which has an annual interest rate of 6.5% compounded annually. How much is in the account at the end of 8 years if the interest is left in the account each year?
		9.4.2	The population of a small town in North Carolina is 2500 people. If it is growing at 3% annually, what is the anticipated population in five years?

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 9: The learner will solve problems involving logarithmic and exponential functions.

OBJECTIVES	SAMPLE MEASURES
9.5 Apply the definition of logarithms.	9.5.1 Find $\log_2 32$. 9.5.2 If $\log_9 x = -2$, find x.
9.6 Use properties of logarithms and exponents.	9.6.1 Which best approximates 5? (a) $\log_2 49$ (b) $\log_2 27$ (c) $\log_2 36$ (d) $\log_2 30$ 9.6.2 Given that $\log 8.1 = 0.9085$, give the value of $\log 8100$. 9.6.3 Solve: $\log_a x - \log_a 3 = \frac{1}{2}\log_a 16$ (a) {12} (b) {11} (c) {24} (d) {7} 9.6.4 Solve: $\log_a(x+3) + \log_a(x-2) = \frac{3}{2}\log_a 9$.
9.7 Use logarithms to solve expressions of the form $a \cdot b^x = c$ for x.	9.7.1 If $2^{x+1} = 50$, then: (a) $x = 24$ (b) $4 < x < 5$ (c) $5 < x < 6$ (d) $6 < x < 7$ 9.7.2 Solve the equation $50 \cdot (1.06)^x = 65$. 9.7.3 Over the past few years college tuition has increased 10% each year. If this continues, when will the tuition at a local university increase from \$3,000 to \$6,000 (i. e., double)?

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 10: The learner will solve problems involving sequences and series.

OBJECTIVES	SAMPLE MEASURES
10.1 Generate the terms of an arithmetic series by iteration.	10.1.1 The first term of a sequence is 3 and each term is 2 more than the previous term. Find the value of the tenth term. Give a general iterative rule for the *n*th term of this sequence.
	10.1.2 If $a_0 = 5$ and $a_{n+1} = a_n + 3$, find the value of a_7.
10.2 Use a calculator or computer to generate the terms of a geometric series by iteration.	10.2.1 If the first term of a series is 0.01 and each term after the first is twice the term before it, what is the tenth term of the sequence? Give a general rule for the *n*th term of this sequence.
	10.2.2 If $a_0 = 1$ and $a_{n+1} = -1.1$, then what is the value of a_3?
10.3 Use summation notation to describe the sums in a series.	10.3.1 What is the value of: $\displaystyle\sum_{n=1}^{4} n+2$ (a) 10 (b) 18 (c) 12 (d) 6
	10.3.2 Write the infinite geometric series $\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+\ldots$ in summation notation.
	10.3.3 Write the series $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots\frac{1}{10}$ in summation notation.

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 11: The learner will solve problems involving probability.

OBJECTIVES	SAMPLE MEASURES
*11.1 Find the probabilities of independent events by multiplying probabilities.	11.1.1 NASA plans to land a space craft on Mars. To do so successfully, all systems at the Earth headquarters and all systems on the satellite must work correctly. The probabilities of each of these working correctly are .95 and .9, respectively. What is the probability that the mission will be completed successfully and the craft will land on Mars?
	11.1.2 Three coins are flipped. What is the probability that there will be exactly two heads showing?
	11.1.3 Name two independent events.
*11.2 Find the probabilities of mutually exclusive events by adding probabilities.	11.2.1 A number wheel is divided into 16 congruent sectors, numbered 1-16. You spin the wheel. What is the probability that the spinner stops on a number which is less than 3 or more than 12?
	11.2 2 A deck of cards consists of 50 cards numbered 1 through 50. What is the probability that a card drawn at random is a multiple of 5 or a multiple of 11?

MATHEMATICS

Algebra II

GRADE LEVEL: 9-12

COMPETENCY GOAL 11: The learner will solve problems involving probability.

<table>
<tr><th>OBJECTIVES</th><th>SAMPLE MEASURES</th></tr>
<tr><td>11.3 Use tree diagrams to model situations involving both the multiplication and addition principles.</td><td>11.3.1 A bag contains 12 marbles, 4 red and 8 green. A child takes the marbles out of the bag one at a time and places them on the floor. Use a tree diagram to help you find the probability that the second marble taken from the bag is green.</td></tr>
<tr><td></td><td>11.3.2 A certain disease has infected a random distribution of 1% of the people in a population. The test for this disease is 95% accurate, meaning (a) a person with the disease will test positive 95% of the time, and (b) a person without the disease will test negative 95% of the time. What is the probability that someone who tests positive actually has the disease?</td></tr>
<tr><td>11.4 Understand the meaning of probabilistic statements.</td><td>11.4.1 An announcer stated that the odds of rain are 2 to 5. What is the probability of rain?</td></tr>
<tr><td></td><td>11.4.2 Explain the meaning of the following statement: "There is a 40% chance of rain tomorrow."</td></tr>
<tr><td></td><td>11.4.3 A teacher announces the following distribution of grades on a recent test:
<table>
<tr><th>Grade</th><th>Number of Students</th></tr>
<tr><td>A</td><td>7</td></tr>
<tr><td>B</td><td>6</td></tr>
<tr><td>C</td><td>8</td></tr>
<tr><td>D</td><td>4</td></tr>
<tr><td>F</td><td>2</td></tr>
</table>
If a student from the class is chosen at random, what is the probability that the student made a B?</td></tr>
</table>

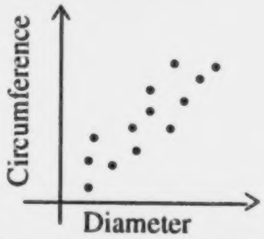
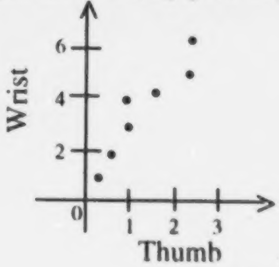
NOTES

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 1: The learner will model real-world phenomena using techniques of data analysis.

OBJECTIVES		SAMPLE MEASURES	
1.1	Recognize a mathematical model of linear, quadratic, exponential, trigonometeric, and logarithmic functions.	1.1.1	Below is a scatter plot of ordered pairs gathered by measuring the circumference and diameter of a circle. What type of function best models these data? 
		1.1.2	Describe how the physical relationship in a square is represented by the model $A = s^2$.
1.2	Use a scatter plot to determine if a given model is appropriate.	1.2.1	In *Gulliver's Travels*, the Lilliputians measured the circumference of Gulliver's thumb to predict the circumference of his wrist. They estimated that twice around the thumb is equal to once around the wrist. The scatter plot of the members of a class who measured their thumbs and wrists is provided below. Is the estimate provided by the Lilliputians appropriate? Justify your answer. 

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 1: The learner will model real-world phenomena using techniques of data analysis.

OBJECTIVES

1.3 Find the equation of a line that models a bivariate linear data set.

SAMPLE MEASURES

1.3.1 The following data were collected regarding the number of hours some students spent studying and their grade on a test:

hours	2.3	1.7	5.6	3.2	2.1	3.3	0.5	1.1	1.9
grade	81	83	93	95	88	91	60	80	85

Find a model for these data.

OBJECTIVES

1.4 Use the linear least squares method to fit linear data.

SAMPLE MEASURES

1.4.1 Find the linear model that best fits the following data which were collected about students' study time and grade on test:

Hours	Grade
2.3	81
1.7	83
5.6	93
3.2	95
2.1	88
3.3	91
0.5	60
1.1	80
1.9	85

Use your calculator to find the model for this data using linear least squares.

1.4.2 In a nine week course, the final grades (A, B, C, D, and F) were given the scores $y = 4, 3, 2, 1$ and 0 respectively. The least-squares line for predicting scores from x, the number of absences, was $y = 3.5 - 0.5x$. What letter grade can you predict for a student with (a) one absence? (b) 3 absences? (c) 8 absences?

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 1: The learner will model real-world phenomena using techniques of data analysis.

OBJECTIVES	SAMPLE MEASURES
1.5 Interpret the slope and y–intercept of a linear model.	1.5.1 The following data were collected by counting the number of cricket chirps per minute at different temperatures:

chirps	56	92	110	135	159	175	190	200
temp.	55	60	65	70	77	81	84	88

The model for this data is:

temperature = .033 x chirps + 45.

What is the meaning of the slope? What is the meaning of the y–intercept?

1.5.2 A group of students investigated the relationship between a student's score on a standardized test and their beginning yearly salaries on full-time jobs. The model they developed is:

Salary = 25 • (test score) – 6250.

What is the meaning of the slope? Does the y-intercept in this model have meaning? Explain.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 1: The learner will model real-world phenomena using techniques of data analysis.

OBJECTIVES | **SAMPLE MEASURES**

1.6 Determine the goodness-of-fit of a model using residuals and/or correlation coefficients.

1.6.1 A data set was collected by measuring the armspan and the height of students (in inches). How good is the line $y = x$ as a model for this data? Explain.

Armspan	Height
64	62
69	70
62	62
63	60
70	72
71	70.5
69	68
65	67.5

1.6.2 According to the model above, what is the residual associated with the point (63, 60)?

1.7 Use models where appropriate to draw conclusions or make predictions.

1.7.1 A model for how life expectancy depends on the year of birth is:

life expectancy = 0.42 x birthyear − 761

Estimate the life expectancy for a person born in the year 2000. Estimate for the year 1850.

1.7.2 The price per pound of apples is described by the fraction

$$\text{Price} = 10 \sin\left(\frac{\pi}{6} \cdot t\right) + 99,$$

where t represents the number of months since January. Estimate the price per pound of apples in March. According to the model, what happens to the price of apples over the year? The next year?

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 2: The learner will graph, transform, use as mathematical models, and compose basic functions.

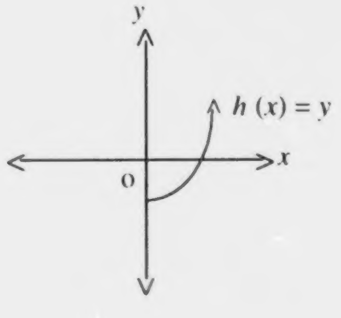
OBJECTIVES		SAMPLE MEASURES	
2.1	Sketch graphs of the basic functions (constant, linear, quadratic, cubic, square root, absolute value, reciprocal, trigonometric, exponential, logarithmic, and greatest integer).	2.1.1	Use graphs to compare the y-values of $y = x$; $y = \sqrt{x}$; and $y = x^2$ for $0 < x < 1$, and for $x > 1$.
		2.1.2	Which basic function graphs are symmetric about the y-axis? Which are symmetric about the origin?
		2.1.3	Identify the domain and the range of each basic function.
2.2	Compare information given by local behavior versus global behavior on graphs produced on an automatic grapher.	2.2.1	Graph $y = x^3 - x$ on the graphing calculator using the viewing window $-5 < x < 5$. Compare with the graph of the same function on $-20 < x < 20$.
		2.2.2	Compare the graphs of $f(x) = x^2$ and $g(x) = 2^x$ by looking at both on the intervals $x \in [-2, 2]$ and $x \in [2, 8]$.
2.3	Find the domain of a function.	2.3.1	What is the domain of $f(x) = \frac{1}{x^2 - k}$?
		2.3.2	Identify the domain of $y = \sqrt{\frac{x^2 - 9}{x + 1}}$.
		2.3.3	Determine an appropriate domain for the function $y = \sqrt{2x - 9}$ using an automatic grapher.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 2: The learner will graph, transform, use as mathematical models, and compose basic functions.

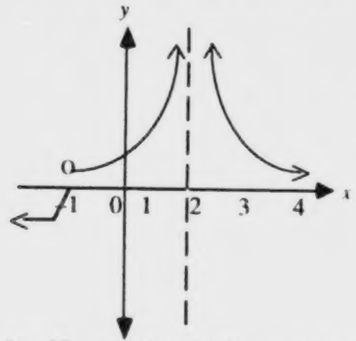
OBJECTIVES		SAMPLE MEASURES	
2.4	Estimate the range of a function.	2.4.1	Using an automatic grapher, estimate the range of $y = \frac{x}{x-1}$.
		2.4.2	Compare the ranges of the functions $y = \sqrt{3x+1}$ and $y = \sqrt{3x} + 1$.
2.5	Determine the symmetry of a given graph.	2.5.1	Determine whether the following functions have graphs that are even, odd, or neither: $y = \sin(x) + x$, $y = \sin(x^2)$, $y = x^3 - 5x + 1$.
		2.5.2	Given the first and fourth quadrant portion of the graph of $y = h(x)$, graph the second and third quadrant portion if $h(x) = -h(-x)$. 

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 2: The learner will graph, transform, use as mathematical models, and compose basic functions.

OBJECTIVES	SAMPLE MEASURES
2.6 Identify continuous and discontinuous functions and locate points of discontinuity.	2.6.1 What are the points of discontinuity for the function shown below? 
	2.6.2 Identify point(s) of discontinuity for the function $y = \frac{1}{x-3}$.
	2.6.3 Show a function which has points of discontinuity at $x = 0$ and $x = 2$.
	2.6.4 Using an automatic grapher, graph: $y = \frac{\sin x}{x}$. Identify any points of discontinuity. Do the same for: $y = \frac{1}{x+3}$. Write a short paragraph to a student in next year's class about the use of an automatic grapher and points of discontinuity.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 2: The learner will graph, transform, use as mathematical models, and compose basic functions.

OBJECTIVES	SAMPLE MEASURES
2.7 Graph transformations and combinations of transformations for all basic functions.	2.7.1 Sketch the graph of the following functions: (a) $y = \sqrt{x} - 2$ (b) $y = \frac{1}{x-3}$ (c) $y = \sin(2x)$ (d) $y = \frac{1}{2} \cdot \lvert x \rvert$ (e) $y = \frac{1}{2} \cdot \lvert x - 1 \rvert$ 2.7.2 Explain how the graph of $y = 2 \cdot f(x + 3)$ compares to $y = f(x)$.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 2: The learner will graph, transform, use as mathematical models, and compose basic functions.

OBJECTIVES	SAMPLE MEASURES
2.8 Find coordinates of zeros and maximum or minimum points of a given function.	2.8.1 The path of a projectile follows the function $h(t) = -4(t-2)^2 + 3\sqrt{t+3} + 14$. Use an automatic grapher to find the maximum height of the projectile and how long the projectile will remain in the air.
	2.8.2 A submarine travels along the path of $y = \frac{1}{x}$. A ship is anchored at (3, 3) on the coordinate plane. At what point is the submarine closest to the ship?
2.9 Write the equation of a function from a description of its behavior.	2.9.1 A pup tent is made by folding a retangular 8.5 by 11 inch paper in half. Develop a function to describe the volume of the tent in terms of the height of the pole. Explain any special questions or conditions of your fold or your function.
2.10 Solve inequalities using an automatic grapher or number line analysis.	2.10.1 Look at a graph of $y = x^3 - 2x^2 - 5x + 6$. Use the graph to solve $x^3 - 2x^2 - 5x + 6 \geq 0$.
	2.10.2 Use number line analysis to solve $(x-1)^2(x+2)(x-3) < 0$.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 2: The learner will graph, transform, use as mathematical models, and compose basic functions.

OBJECTIVES	SAMPLE MEASURES
2.11 Compose two functions and find the domain of the composition.	2.11.1 Suppose $f(x) = \sqrt{x+1}$ and $g(x) = \frac{x}{x-2}$. The domain of $f \circ g$ is: (a) $x \geq -1$ (b) $x \neq 2$ (c) $x \neq 3$ (d) $x \neq 3, x \geq -1$ 2.11.2 Find the equation of $g \circ f$.
2.12 Analyze a function by decomposing it into simpler functions.	2.12.1 a. Sketch $\frac{1}{x^2 - 6x + 8}$ by first sketching $g(x) = x^2 - 6x + 8$ and then sketching $f(x) = \frac{1}{g(x)}$. b. Sketch $f(x) = \lvert x^2 - 6x + 8 \rvert$ by first sketching $g(x) = x^2 - 6x + 8$ and then sketching $f(x) = \lvert g(x) \rvert$. c. On a single set of axes, sketch the graphs of $f(x) = \sqrt{x}$ and $g(x) = x + 1$. Demonstrate how the graphs of f and g can be used to find the value of (a) $f(g(3))$ and (b) $g(f(4))$. 2.12.2 Express each of the following as a composition of basic functions: $y = \sin^2(3x)$ $y = \ln\sqrt{\frac{1}{x}}$

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 2: The learner will graph, transform, use as mathematical models, and compose basic functions.

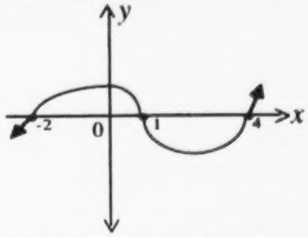
OBJECTIVES	SAMPLE MEASURES
2.13 Find the inverse of a function and the domain of the inverse.	2.13.1 Let $f(x) = \sqrt{x+3} + 2$. What are the domain and the range of $f^{-1}(x)$? (a) $D: x \neq 3$ $R: y \neq 2$ (b) $D: x > -3$ $R: y > 2$ (c) $D: x \geq 2$ $R: y \geq -3$ (d) $D: x \geq -3$ $R: y \geq 2$ 2.13.2 Find the inverse of the function $f(x) = x^3 - 3$. What are the domain and range of $f(x)$? What are the domain and range of $f^{-1}(x)$?

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 3: The learner will graph and use as models polynomial and rational functions.

OBJECTIVES	SAMPLE MEASURES
3.1 Find zeros and factors of polynomials algebraically (where appropriate) and/or using an automatic grapher.	3.1.1 Find the *x*-intercepts of the function $f(x) = x^4 + 5x^3 - 7x^2 + 1$ using an automatic grapher. Estimate to the nearest hundredth.
	3.1.2 Find the *x*-intercepts of the function $f(x) = 1.2x^3 - 3.3x^2 + 1.9x + 1$. Estimate to the nearest tenth.
	3.1.3 Find the zeros of the polynomial $m(x)$ and factor the polynomial $m(x) = x^3 + 4x^2 + x - 6$.
	3.1.4 The graph of the function $g(x)$ is shown below. Give at least two possible equations for $g(x)$. 
3.2 Estimate the turning points of polynomial functions using an automatic grapher.	3.2.1 Given the following function: $y = x^4 - 3x^2 + x - 4$, estimate the turning points of the function to the nearest tenth.
	3.2.2 A box is formed from a sheet of $8\frac{1}{2}$" x 11" notebook paper by cutting the same size square from each corner of the paper and folding the paper to form a box. Write a function which expresses the volume and then determine the largest volume.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 3: The learner will graph and use as models polynomial and rational functions.

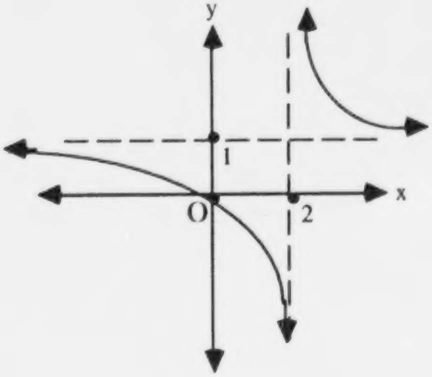
OBJECTIVES	SAMPLE MEASURES
3.3 Find the zeros and vertical asymptotes of a rational function through analysis of the polynomials in the numerator and denominator.	3.3.1 Find zeros of the function: $f(x) = \frac{\lvert x^3 \rvert + 1}{x^3 - x + 2}$.
	3.3.2 Defend or contradict the following statement: *The function* $f(x) = \frac{x + 2}{x^2 - 4}$ *has vertical asymptotes at* $x = 2$ *and at* $x = -2$.
	3.3.3 Find the zeros and the vertical asymptotes of the function: $y = \frac{(x - a)^2}{(x + b)(x + c)}$
3.4 Find the horizontal asymptote of a rational function by comparing the growth of the polynomials in the numerator and denominator.	3.4.1 (a) For large x-values compare the y-values of the two functions $y = x^2 - 5$ and $y = x^2 + 1$. (b) For the function $q(x) = \frac{x^2 - 5}{x^2 + 1}$ estimate the range values of $q(x)$ for large x-values. What behavior do you expect in the graph of $q(x)$ for large x-values?

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 3: The learner will graph and use as models polynomial and rational functions.

OBJECTIVES	SAMPLE MEASURES
3.5 Sketch the graph of a rational function labeling the horizontal and vertical asymptotes and the x- and y-intercepts.	3.5.1 Graph the following function: $$r(x) = \frac{x^2 - 4x + 7}{2x^2 - 8}$$ Label intercepts and asymptotes, if any.
	3.5.2 Write an equation for the function whose graph is shown below. 

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 4: The learner will graph, transform, and solve problems involving the exponential and logarithmic functions.

OBJECTIVES		SAMPLE MEASURES	
4.1	Model growth using transformations of: $f(x) = ab^x$ and $f(x) = (1 + r)^x$.	4.1.1	Sketch graphs showing how the balance changes over time in an account that begins with a balance of $500 and earns an annual interest rate of 5%, 7%, or 10%. Use the graph to determine the number of years it takes for the $500 to become $1000.
		4.1.2	The population of algae in a small tank is modeled by $P(t) = 0.02e^{0.017t}$, where $P(t)$ represents the present volume of algae and t the number of months. When will the population double?
4.2	Define and use the logarithmic function as the inverse of the exponential function.	4.2.1	Find the inverse of $f(x) = 3^{x-1} + 2$.
		4.2.2	A bacteria culture is growing continuously at a rate of 5% each hour. The function that models this growth is $P = P_0e^{.05t}$ where P_0 is the initial population. Use the function to find how long it will take for the population to double.
4.3	Find the domain of exponential and logarithmic functions.	4.3.1	For what values of x is the function $g(x) = \ln(\sqrt{x-1} - 4)$ defined?
		4.3.2	Compare the range of $f(x) = e^{2x+1}$ with the domain of $g(x) = \frac{1}{2}\ln(x-1)$.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 4: The learner will graph, transform, and solve problems involving the exponential and logarithmic functions.

OBJECTIVES	SAMPLE MEASURES
4.4 Sketch and investigate the graphs of composition of exponential and logarithmic functions with each other and with other basic functions.	4.4.1 Given the function $m(x) = x^2 + 1$, describe the range of values of the function $y = e^{m(x)}$. 4.4.2 Compare the graphs of the functions $f(x) = \log 10^{(x^2 - 10)}$ and $g(x) = 10^{\log(x^2 - 10)}$.
4.5 Solve equations containing exponential and logarithmic functions using real world problems.	4.5.1 In 1990 the population of Asia was 3.1 billion with a 1.9% annual growth rate. The population of Central and South America was 295 million with an annual growth rate of 2.1%. Will the population of the two continents ever be equal? If so, when? 4.6.2 Helen, being honored for bravery, was given a choice of two awards: (a) \$2000 in an account paying 7% annually for ten years, or (b) \$100 for the first six months with the award doubling every six months for ten years. Which should she choose?

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 5: The learner will use techniques of data analysis to model nonlinear data from real world phenomena.

<table>
<tr><th>OBJECTIVES</th><th>SAMPLE MEASURES</th></tr>
<tr>
<td>5.1 Linearize data using concepts of composition and inverses in order to find a model for data.</td>
<td>5.1.1 The world population (in millions) over time is shown with the following data:
<table>
<tr><th>Year</th><th>Population</th></tr>
<tr><td>1000</td><td>200</td></tr>
<tr><td>1650</td><td>545</td></tr>
<tr><td>1750</td><td>728</td></tr>
<tr><td>1800</td><td>906</td></tr>
<tr><td>1850</td><td>1171</td></tr>
<tr><td>1900</td><td>1608</td></tr>
<tr><td>1950</td><td>2517</td></tr>
<tr><td>1987</td><td>5000</td></tr>
</table>
Linearize the data and find a line through the linear data.

5.1.2 Convert this line to a function that will pass through the original data.</td>
</tr>
</table>

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 5: The learner will use techniques of data analysis to model nonlinear data from real world phenomena.

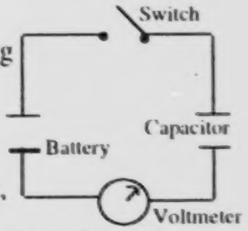
OBJECTIVES

5.2 Rewrite the linear equation that models linearized data to fit the original curved data.

SAMPLE MEASURES

5.2.1 The equation $y = .32x - 1.52$ was found to fit data that had been linearized by plotting $(x, \ln y)$. What equation will fit the original data (x, y)?

5.2.2 An electronic circuit containing a capacitor has voltage 1.52V. When the electrical circuit is broken, the capacitor gradually discharges. At various times t (in minutes) after the circuit is broken, the voltage V (in volts) is read on a voltmeter. The table below shows the data obtained:



t	0	0.5	1.0	1.5	2.0	2.5	3.0
V	1.52	1.25	1.01	0.82	0.67	0.54	0.45

a. Plot $(t, \log V)$. Find an equation of the resulting straight line in the form: $\log V = mt + k$.

b. Solve the equation from part (a) for V in terms of t.

5.2.3 The equation $y = 2.00x - 1.39$ was found to fit data that had been linearized by plotting $(\ln x, \ln y)$. What equation will fit the original data (x, y)?

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 5: The learner will use techniques of data analysis to model nonlinear data from real world phenomena.

OBJECTIVES

5.3 Discuss the goodness-of-fit of the model used to represent data.

SAMPLE MEASURES

5.3.1 The model developed to describe the world population in 5.1.1 is

$$p(y) = 142.4e^{.00274y},$$

where y = year and $p(y)$ = population of that year. Plot the data and the function $p(y)$ on the same axes. Discuss the quality of this model. Include the meaning of the intercept, the coefficient of y, the residuals, and the ability to predict based on the model.

5.3.2 The following table gives the population size, P, of bacteria at various regular time intervals, t.

t	0	1	2	3	4	5	6	7
P	1	2	4	8	12	21	37	61

a. What model do you think will best fit the data? Explain.
b. Find an equation that models the data above?
c. Predict the value population size when $t = 10$.

5.3.3 A set of data has correlation values of +0.986 for (x, ln y) and +0.956 for (ln x, ln y). Which type of function fits the data best, exponential or polynomial?

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 5: The learner will use techniques of data analysis to model nonlinear data from real world phenomena.

OBJECTIVES	SAMPLE MEASURES
5.4 Use log-log re-expression to linearize data of the form $y = a \bullet x^b$ by plotting (log *x*, log *y*).	5.4.1 The following data were taken from the scale given on the Alpo bag to determine the amount of food to give a dog of certain weight:

Weight	Cups
8	1
19	2
37	3
57	4
78	5
103	6
130	7
158	8
190	9

What is Alpo's model for determining the number of cups of food to give a dog of a specific weight?

5.4.2 You are able to fit the line $y_1 = .3x_1 + 4$ to a set of data that has been re-expressed by taking the log of both the *x*–values and the *y*–values (log *x*, log *y*). An approximate model for the original data (*x*, *y*) is

(a) $y = .3x + 4$

(b) $y = 10{,}000x^{.3}$

(c) $y = .3x^4$

(d) $y = 4(.3)^x$

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 6: The learner will graph and transform trigonometric functions, solve trigonometric equations and inequalities, and use trigonometric functions as mathematical models.

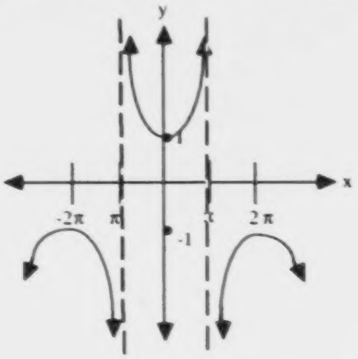
OBJECTIVES	SAMPLE MEASURES
6.1 Use trigonometric functions to model periodic phenomena.	6.1.1 A diving board oscillates 20 cm above and 20 cm below its equilibrium position. It takes one second to move from its highest to its lowest position. Sketch a graph showing time on the horizontal axis and displacement on the vertical axis.
	6.1.2 When you stop at a traffic light a pebble lodges in your tire. As you travel, the distance from the ground to the pebble is a sinusoidal function of time. If the radius of your tire is 13 inches what is a function that represents height of the pebble versus distance traveled?
6.2 Express the tangent, cotangent, secant, and cosecant functions in terms of sine and cosine.	6.2.1 Express each of the following in terms of sine and cosine: (a) $\tan x \bullet \sec^2 x$ (b) $\dfrac{\cot x}{\csc x}$
6.3 Sketch a graph of each of the six trigonometric functions and identify the period of each.	6.3.1 Sketch a graph of each function. Identify the period of each: (a) $y = \tan x$ (b) $y = \csc x$

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 6: The learner will graph and transform trigonometric functions, solve trigonometric equations and inequalities, and use trigonometric functions as mathematical models.

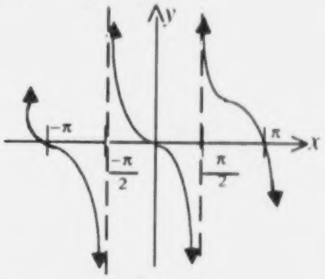
OBJECTIVES	SAMPLE MEASURES
6.4 Recognize and graph transformations of each of the six trigonometric functions.	6.4.1 Write an equation in terms of the secant function for the given graph.  6.4.2 Sketch a graph of $y = -\cot(x - \frac{\pi}{2})$. 6.4.3 The average daily temperature in Hillsborough, North Carolina is given by the function: $\text{temp} = 20 \cos(\frac{2\pi}{365}) + 60.$ What is the meaning of the amplitude?

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 6: The learner will graph and transform trigonometric functions, solve trigonometric equations and inequalities, and use trigonometric functions as mathematical models.

OBJECTIVES	SAMPLE MEASURES
6.5 Use graphs to develop, recognize, and validate trigonometric identities.	6.5.1 The figure below shows the graph of $y = \tan(-x)$.  Write an equation in terms of cotangent that has the same graph. What identity have you demonstrated?
	6.5.2 Use an automatic grapher to graph $y = \csc^2 x - \cot^2 x$. What identity is suggested by your graph?
	6.5.3 Use graphs to demonstrate that $\cos 2x \neq \cos x + \cos x$. What conclusions can you draw from this?

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 6: The learner will graph and transform trigonometric functions, solve trigonometric equations and inequalities, and use trigonometric functions as mathematical models.

OBJECTIVES	SAMPLE MEASURES
6.6 Understand how the values of sine and cosine are represented on the unit circle.	6.6.1 A calculator in radian mode shows that $\sin 4 = -0.7568$ and $\cos 4 = -0.6536$. Sketch and label a unit circle to show how these values are related to each other and to 4.
	6.6.2 Name all angles between 0^0 and 360^0 whose cosine is equal to $\cos 52^0$.
	6.6.3 Use the unit circle to explain $\sin^2 (4) + \cos^2 (4) = 1$.
6.7 Use the symmetry of the unit circle to develop, recognize, and validate trigonometric identities.	6.7.1 Use the unit circle to explain why $\cos (\pi - \emptyset)$ is equal to $\cos (\pi + \emptyset)$ for all $\emptyset$.
	6.7.2 Which of the following is always equal to $\sin \emptyset$? (a) $\sin (-\emptyset)$ (b) $\sin (\pi + \emptyset)$ (c) $\cos (\frac{\pi}{2} + \emptyset)$ (d) $\sin (\pi - \emptyset)$
6.8 Understand the relationship between trigonometry in degree mode and trigonometry in radian mode.	6.8.1 Make a sketch on the unit circle to illustrate the facts that $\sin (1 \text{ radian}) = 0.8415$ and $\sin (1 \text{ degree}) = 0.01745$.
	6.8.2 The point $(\frac{\pi}{2}, 1)$ is on the graph of $y = \sin x$ because: (a) $\sin 1^0 = \frac{\pi}{2}$ (b) $\sin \frac{\pi^0}{2} = 1$ (c) $\sin \frac{\pi}{2}$ radians $= 1$ (d) $\sin 1$ radian $= \frac{\pi}{2}$

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 6: The learner will graph and transform trigonometric functions, solve trigonometric equations and inequalities, and use trigonometric functions as mathematical models.

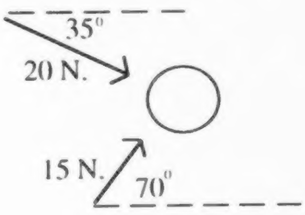
OBJECTIVES		SAMPLE MEASURES	
6.9	Find the radian measure that corresponds to a given angle or arc length.	6.9.1	In a unit circle, a central angle of 2 radians intercepts an arc of what length?
		6.9.2	What is the radian measure of an angle that intercepts an arc of length T in a circle of radius 3?
6.10	Solve trigonometric equations and inequalities either algebraically or using an automatic grapher.	6.10.1	Solve $\tan(4x) = 1, 0 \le x \le 2\pi$.
		6.10.2	Solve $3 \bullet \cos(x + \pi) = 2$.
		6.10.3	Find the solutions of $\sin(2x + \pi) > 0.45$ accurate to the nearest hundredth.
6.11	Find values of inverse trigonometric functions, applying appropriate domain and range restrictions.	6.11.1	Identify the domain and the range of the function $y = 2 \bullet \sin^{-1}(2x)$.
		6.11.2	Describe one way to restrict the domain of $f(x) = \sec x$ so that $f^{-1}(x)$ is a function.
		6.11.3	Graph $y = \cos^{-1}(x)$.
		6.11.4	Use a calculator to evaluate $\cot^{-1}(0.8)$. Sketch a graph of $y = \cot^{-1}(x)$ and show the point on the graph that corresponds to this value.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 6: The learner will graph and transform trigonometric functions, solve trigonometric equations and inequalities, and use trigonometric functions as mathematical models.

OBJECTIVES	SAMPLE MEASURES
6.12 Evaluate and graph compositions of trigonometric and inverse trigonometric functions.	6.12.1 For what values of x is $\tan^{-1}(\tan x) = x$? (a) all real numbers (b) $0 \le x \le \pi$ (c) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ (d) $-\frac{\sqrt{2}}{2} \le x \le \frac{\sqrt{2}}{2}$ 6.12.2 Express $\sin(\tan^{-1} \emptyset)$ in terms of $\emptyset$.
6.13 Use the Law of Sines and Law of Cosines to solve problems involving triangles and vectors.	6.13.1 Two airplanes take off from an airport at the same time. Plane A travels N 32^0 E at a constant speed of 380 mph. Plane B travels at a constant speed of 420 mph in the direction S 49^0 W. How far apart will the planes be if they maintain their constant speeds for 1.5 hours? 6.13.2 Two volleyball players hit the ball at the same time. The forces they exert are represented by the arrows drawn below. Describe the vector representing the total force exerted on the ball. 

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 6: The learner will graph and transform trigonometric functions, solve trigonometric equations and inequalities, and use trigonometric functions as mathematical models.

OBJECTIVES	SAMPLE MEASURES
6.14 Convert complex numbers from rectangular form to polar form.	6.14.1 Which of the following expresses $-1 + i$ in polar form? (a) $\sqrt{2}\ (\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$ (b) $\sqrt{2}\ (\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$ (c) $\sqrt{2}\ (\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4})$ (d) $\sqrt{2}\ (\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3})$ 6.14.2 Express $3 - i\sqrt{3}$ in polar form.
6.15 Convert complex numbers from polar form to rectangular form.	6.15.1 Which of the following expresses $4(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6})$ in rectangular form? (a) $-2\sqrt{3} - 2i$ (b) $2\sqrt{3} + 2i$ (c) $2 + 2i\sqrt{3}$ (d) $-2 - 2i\sqrt{3}$ 6.15.2 Express $2\ (\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4})$ in rectangular form.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 6: The learner will graph and transform trigonometric functions, solve trigonometric equations and inequalities, and use trigonometric functions as mathematical models.

OBJECTIVES	SAMPLE MEASURES
6.16 Use DeMoivre's Theorem to find roots and powers of complex numbers.	6.16.1 Solve $x^3 + 1 = 0$ for all roots.
	6.16.2 Use DeMoivre's Theorem to evaluate: $(-\sqrt{3} + i)^{10}$.
6.17 Use an automatic grapher to explore polar equations.	6.17.1 Use your graphing calculator to describe the graph of $r = 4 \cos 2 \phi$.
	6.17.2 Use your graphing calculator to describe the graph of $y = 1 - \sin \phi$.
	6.17.3 Use an automatic grapher to graph $r = -3$ and $r = 3\cos 2\phi$. Discuss the points of intersection.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 7: The learner will use discrete mathematics concepts to solve problems.

OBJECTIVES	SAMPLE MEASURES
7.1 Follow an algorithm.	7.1.1 The following algorithm generates a sequence of numbers. 1. Start with any positive integer k. 2. If $k = 4$, then STOP. 3. If k ends in 4 (the unit digit is 4), then cross out the 4 in the units place and go to step 2. For example, if k is 5494, then change k to 549 and go to step 2. 4. If k ends in 0, (the units digit is 0), then cross out the 0 in the units place and go to step 2. 5. Double k and go to step 2. Starting with $k = 154$, list the sequence of numbers the algorithm generates until it stops.
	7.1.2 A polynomial function $f(x)$ has values $f(2) = -5$, $f(5) = 3$, and $f(8) = 10$. Between which two x-values is $f(x)$ guaranteed to have a root? Using the bisection algorithm, what x-value would you suggest to guess for the root?

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 7: The learner will use discrete mathematics concepts to solve problems.

OBJECTIVES	SAMPLE MEASURES
7.2 Use operations with matrices and the inverse of a matrix to solve real world problems.	7.2.1 In a school with grades 10, 11, and 12, there are 152 females and 131 males in the tenth grade, 137 females and 140 males in the eleventh grade, and 121 females and 111 males in the twelfth grade. A scholarship program allows the school to nominate 5% of the females and 3% of the males. If the administration distributes the nominations equally across each grade, how many tenth, eleventh, and twelfth graders will be nominated? 7.2.2 A species of insects lives for 12 weeks. In the matrix below, the first row represents the birth rate of the insect for each age group. Succeeding rows show the survival rate for each age group. Find the long-term population distribution. After how many cycles does the total population reach 1000? $\begin{array}{cc} & \begin{array}{cccc} 0\text{–}3 & 3\text{–}6 & 6\text{–}9 & 9\text{–}12 \end{array} \\ L = & \begin{bmatrix} .5 & 2.7 & 4.8 & .7 \\ .3 & 0 & 0 & 0 \\ 0 & .6 & 0 & 0 \\ 0 & 0 & .4 & 0 \end{bmatrix} \end{array}$ Initially, the population distribution of the insect is given in the matrix $\begin{bmatrix} 30 \\ 25 \\ 70 \\ 10 \end{bmatrix}$

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 7: The learner will use discrete mathematics concepts to solve problems.

OBJECTIVES	SAMPLE MEASURES
7.2 Use operations with matrices and the inverse of a matrix to solve real world problems.	7.2.3 Students took data on the weather. They found that $\frac{1}{4}$ of the time a sunny day is followed by a cloudy day. The students also discovered that $\frac{5}{7}$ of the time a cloudy day is followed by a sunny day. Represent this information as a transition matrix in a Markov chain.
7.3 Use permutations and combintions to solve problems.	7.3.1 Five students are to be seated in a row of five chairs. In how many ways might this be accomplished if: (a) two of these students must always be seated together? (b) two of these students must never be together? (c) the row is changed to a circle? (d) a sixth student is included? (e) the row contained ten chairs?
	7.3.2 Generalize the different possibilities in 7.3.1 with a mathematical model.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 7: The learner will use discrete mathematics concepts to solve problems.

OBJECTIVES

7.4 Classify events as independent or dependent or mutually exclusive.

SAMPLE MEASURES

7.4.1 Two dice are tossed. Classify the following pairs of events as independent, mutually exclusive, or neither.

(a) One die shows 2. Another die shows 3.
(b) One die shows 2. The same die does not show 3.
(c) One die shows 2. The same die shows 3.

7.4.2 An auto manufacturer buys plastic molded parts from two companies. A frequency distribution of parts by quality that the auto manufacturer has received is:

	A	B
excellent	60	24
acceptable	272	20
unacceptable	16	8

(a) What is the probability that a randomly selected part is from A?
(b) What is the probability that a randomly selected part is excellent?
(c) What is the probability that a randomly selected part is both from A and is excellent?
(d) Are the quality of parts and their sources independent? Include an example in your explanation.

7.4.3 Explain in words why, if two events can happen and they are mutually exclusive, then they cannot be independent. Include an example with your explanation.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 7: The learner will use discrete mathematics concepts to solve problems.

OBJECTIVES	SAMPLE MEASURES

7.5 Interpret data in terms of mean and standard deviation.

7.5.1 John made an 85 on a math test and an 85 on a science test. The class average on both tests is 75. Suppose the standard deviation for the math and science tests are 5.6 and 11.8 respectively. Compare John's performance on the two tests. Is he doing equally well in the two courses?

7.5.2 Twenty puppies were taught to sit and stay on command. Find the mean and standard deviation of the number of trials required before they learned to do this.

no. of trials	7	8	9	10	11	12
no. of puppies	1	2	5	4	4	4

If five more puppies were taught, each taking 7 trials, how would this affect the mean and standard deviation? Explain why this happens.

7.5.3 Is it possible for a set of data to have a variance of 0? Explain.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 7: The learner will use discrete mathematics concepts to solve problems.

OBJECTIVES

7.6 Recognize and use the normal distribution curve to interpret data.

SAMPLE MEASURES

7.6.1 The lengths of 2400 babies born at Valley Hospital in the past year were normally distributed. The mean was 50.5 cm and the standard deviation was 3.6 cm.

(a) What percent of babies' lengths were between 46.9 cm and 50.8 cm?

(b) Find the interval about the mean that includes 95% of the babies' lengths.

7.6.2 The following table gives weekly wages of city employees:

Wages	\$230-\$240	\$240-\$250	\$250-\$260	\$260-\$270	\$270-\$280	\$280-\$290	\$290-\$300
Frequency	11	24	30	10	13	8	4

Are these wages normally distributed? Explain your answer.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 7: The learner will use discrete mathematics concepts to solve problems.

OBJECTIVES	SAMPLE MEASURES
7.7 Find indicated terms in sequences.	7.7.1 What is the 29th term of the arithmetic sequence $-7, -4, -1, \ldots$? (a) 74 (b) 83 (c) 80 (d) 77
	7.7.2 What is the 19th term of the geometric sequence $1, \frac{1}{2}, \frac{1}{4}, \ldots$?
	7.7.3 Find a sequence which has two geometric means between 1 and 27.
	7.7.4 Find the first term of a sequence whose fourth term is 19 and whose eleventh term is 59.
7.8 Find the sum of a finite series.	7.8.1 What is the sum of the first 11 terms of the series $(-3) + (-1) + 1 + 3 \ldots$?
	7.8.2 Find the sum of the first seven terms of $.5 + .25 + .125 + \ldots$
	7.8.3 If you go back through 11 generations, how may ancestors do you have? Count your parents as the first generation, your grandparents as second, etc.

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 7: The learner will use discrete mathematics concepts to solve problems.

OBJECTIVES	SAMPLE MEASURES
7.9 Find the limit of an infinite sequence.	7.9.1 Find the limit of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{n}{n+1}$.
	7.9.2 Write an infinite sequence whose limit is $\frac{3}{4}$. What is the nth term? Justify $\frac{3}{4}$ as the limit.
	7.9.3 Use appropriate technology (computer or calculator) to approximate the limit of the sequence $\frac{n}{2n+1}$ as n approaches infinity.
7.10 Determine whether a given series converges or diverges.	7.10.1 Which of the following series is convergent? (a) $.5 + .25 + .125 + \ldots.$ (b) $(-5) + (-3) + (-1) + 3 + \ldots$ (c) $1 + 4 + 16 + 64 + \ldots$ (d) All of the above
	7.10.2 What is the sum of the convergent series in 7.10.1? Explain.
	7.10.3 Write an example of a convergent series. Explain. Write an example of a divergent series. Explain

MATHEMATICS

Advanced Mathematics

GRADE LEVEL: 9–12

COMPETENCY GOAL 7: The learner will use discrete mathematics concepts to solve problems.

OBJECTIVES	SAMPLE MEASURES
7.11 Use mathematical induction to establish a generalization.	7.11.1 What are the basic steps that are used to prove by induction that the sum of the first n positive integers is $n(n + 1)/2$?
	7.11.2 In a room with n people (n greater than or equal to 2) each person shakes hands with every other person. Prove that there are $(n^2 - n)/2$ handshakes.
	7.11.3 What is the least number of successful moves that will be required to assemble a 7-piece jigsaw puzzle? an n-piece puzzle? Use mathematical induction to explain why the last move in putting together a 30-piece puzzle is the 29th move.

NOTES

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: Use elementary functions, including algebraic, trigonometric, exponential, and logarithmic.

OBJECTIVES		SAMPLE MEASURES	
1.1	Define a function and relate the idea of function to real situations.	1.1.1	Given that $x^2 + y^2 = 25$ is not a function. Solve for y and write as a union of two functions.
		1.1.2	Create a function to describe the path a rocket might take.
1.2	Find the domain and range of a function with and without an automatic grapher.	1.2.1	Find the domain of $f(x) = \sqrt{x - a}$, where a is a real number. (a) $x > a$ (b) $x \geq a$ (c) $x < a$ (d) $x \leq a$
		1.2.2	Determine the domain of: $y = \sqrt{\dfrac{x-1}{x+1}}$.
		1.2.3	If $y = \sqrt{4 - x^2}$, find the range of the function.
		1.2.4	Find, to the nearest hundredth, the range of $f(x) = x \sin x$ on the interval $[1, 3]$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 1: Use elementary functions, including algebraic, trigonometric, exponential, and logarithmic.

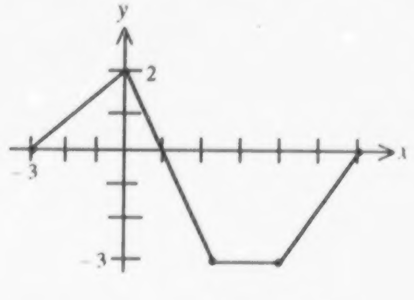
OBJECTIVES	SAMPLE MEASURES
1.3 Find the sum, product, and quotient of two functions.	1.3.1 If $f(x) = 3x - 5$ and $g(x) = -2x + 7$, find $(f + g)(2)$. (a) 4 (b) 0 (c) 3 (d) $x - 2$ 1.3.2 If $f(x) = 3x - 5$ and $g(x) = 2x + 7$, find $(fg)(x)$. (a) $6x^2 + 11x - 35$ (b) $-6x^2 + 31x - 35$ (c) $6x^2 - 31x + 35$ (d) $-6x^2 + 14x - 35$ 1.3.3 If $f(x) = 3x - 5$ and $g(x) = 2x - 4$, find $\left(\frac{f}{g}\right)(2.78)$.
1.4 Find the composition of two functions.	1.4.1 If $f(x) = 5x^2 - 9x + 7$ and $g(x) = 5$, find $g(f(x))$. (a) 87 (b) 5 (c) $25x^2 - 45x + 7$ (d) $25x^2 - 45x + 35$ 1.4.2 If $f(x) = x^2$ and $g(x) = 5x + 3$, find $g \circ f(x)$. 1.4.3 Write two functions that would compose to make $y = ln\,(x^2 + 3x) - 2$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: Use elementary functions, including algebraic, trigonometric, exponential, and logarithmic.

OBJECTIVES	SAMPLE MEASURES
1.5 Find the domain of a composition of two functions.	1.5.1 If $f(x) = \sqrt{2x - 5}$ and $g(x) = x + 5$, find the domain of $f \circ g(x)$. 1.5.2 If $f(x) = ln\ x$ and $g(x) = \sin x$, find the domain of $f \circ g(x)$.
1.6 Find and apply the absolute value of a function.	1.6.1 Given the graph of $f(x)$ below, sketch the graph of $\lvert f(x)\rvert$.  1.6.2 Given the graph of $f(x)$ in 1.6.1 sketch a graph of $f(\lvert x\rvert)$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 1: Use elementary functions, including algebraic, trigonometric, exponential, and logarithmic.

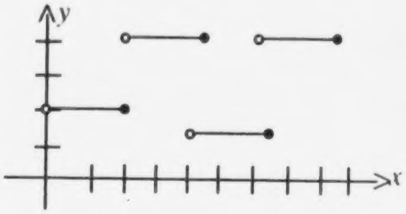
OBJECTIVES	SAMPLE MEASURES
1.7 Find and use the inverse of a function.	1.7.1 Find the inverse of $f(x) = \frac{x}{x-1}$. (a) $f^{-1}(x) = \frac{x}{1-x}$ (b) $f^{-1}(x) = \frac{x-1}{x}$ (c) $f^{-1}(x) = \frac{1-x}{x}$ (d) $f^{-1}(x) = \frac{x}{x-1}$ 1.7.2 If $f(x) = e^{(5x+3)}$, find $f^{-1}(x)$. 1.7.3 Find sin (ARC tan x). (a) $\frac{\sqrt{x^2+1}}{x}$ (b) $\frac{x}{\sqrt{x^2+1}}$ (c) $\frac{\sqrt{x^2-1}}{x}$ (d) $\frac{x}{\sqrt{x^2-1}}$

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: Use elementary functions, including algebraic, trigonometric, exponential, and logarithmic.

OBJECTIVES	SAMPLE MEASURES
1.8 Determine if a function is odd, even, or neither odd nor even.	1.8.1 Which of the following is not an odd function? (a) $y = \sin x$ (b) $y = x^3$ (c) $y = x^3 - x$ (d) $y = x^3 - 1$
	1.8.2 Which of the six basic trigonometric functions are even?
1.9 Determine periodicity and amplitude of a function.	1.9.1 What is the fundamental period of $y = 3\sin(3\pi x) - 1$?
	1.9.2 Find the fundamental period of the function defined by: $f(x) = 3 - 2\cos^2 \frac{\pi x}{3}$ (a) 1 (b) 2 (c) 3 (d) 5
	1.9.3 Determine the period of: 
	1.9.4 Determine the amplitude of the function in 1.9.1.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 1: Use elementary functions, including algebraic, trigonometric, exponential, and logarithmic.

OBJECTIVES	SAMPLE MEASURES
1.10 Describe the symmetry of a function.	1.10.1 Which of the following has a graph that is symmetric with respect to the origin? (a) $y = \frac{x+1}{x}$ (b) $y = -x^5 + 3x$ (c) $y = (x-1)^3 + 1$ (d) $y = x^4 - 2x^2$ 1.10.2 Determine the symmetry of these functions: (a) $y = (\sin x)^2$ (b) $y = \log_{10} x$ (c) $y = x^2 - 3x$
1.11 Find the asymptotes of a function.	1.11.1 If $f(x) = e^x$, which of the following is an asymptote in the graph of $f(x)$? (a) $y = 0$ (b) $x = 0$ (c) $y = x$ (d) $y = -x$ 1.11.2 State all vertical and horizontal asymptotes of: $f(x) = \frac{x^3 - x}{x^3 - 4x}$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 1: Use elementary functions, including algebraic, trigonometric, exponential, and logarithmic.

OBJECTIVES	SAMPLE MEASURES
1.12 Find the zeros of a function.	1.12.1 Determine the zeros of: $f(x) = x^3 + 4x^2 + 4x$. (a) -2 (b) $0, -2$ (c) $0, 2$ (d) 2 1.12.2 Find the zeros of $y = x^4 - 3x^3 + x$. 1.12.3 Find the zeros of $y = 2x^3 - 5x^2 + 5x - 14$.
1.13 Use a^x and $\log_a x$ and their inverse relationships.	1.13.1 Find a when $\log_a 2^a = \frac{a}{4}$. (a) 4 (b) 8 (c) 16 (d) 32 1.13.2 Graph $y = \ln(x - 2)$ and its inverse.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 2: Find and use limits of various functions.

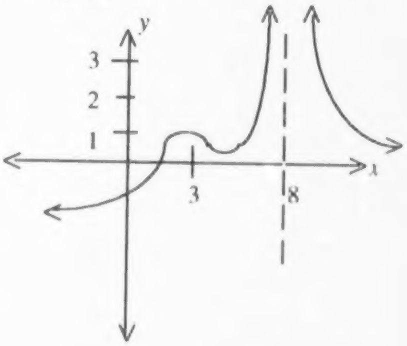
OBJECTIVES		SAMPLE MEASURES	
2.1	Find limits of functions by direct substitution.	2.1.1	Find $\lim\limits_{x \to 2} (x - 5)(x + 3)$. (a) -15 (b) 0 (c) 2 (d) non existent
		2.1.2	Find $\lim\limits_{x \to 1} \left(\frac{x^2 - 4}{x + 5}\right)$.
2.2	Find the limit of a quotient of indeterminant form.	2.2.1	Find $\lim\limits_{x \to 2} \frac{x^2 - 4}{x - 2}$.
		2.2.2	Find $\lim\limits_{h \to 0} \frac{\sqrt{5 + h} - \sqrt{5}}{h}$.
2.3	Find the limit of a function as the independent variable approaches infinity.	2.3.1	Find $\lim\limits_{x \to \infty} \frac{2x^2 - 5x + 7}{x^2 - 3x + 4}$. (a) 0 (b) non-existent (c) 2 (d) ∞
		2.3.2	Find $\lim\limits_{x \to \infty} \left(2 - \frac{5}{x - 2}\right)$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 2: Find and use limits of various functions.

OBJECTIVES	SAMPLE MEASURES
2.4 Find the limit of a function by examining its graph.	2.4.1 Find $\lim_{x \to 3} f(x)$, if it exists.  2.4.2 Find the $\lim_{x \to 8} f(x)$ if it exists. See figure above.
2.5 Recognize the limit of special functions.	2.5.1 Find $\lim_{n \to \infty} (1 + \frac{1}{n})^n$. (a) 1 (b) o (c) e (d) non-existent 2.5.2 Find $\lim_{x \to 0} \frac{\sin x}{x}$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 2: Find and use limits of various functions.

OBJECTIVES	SAMPLE MEASURES
2.6 Recognize that functions have non-existent limits.	2.6.1 Find $\lim_{x \to 0} \sin \frac{1}{x}$ if it exists. (a) 0 (b) 1 (c) ∞ (d) does not exist
	2.6.2 Show by graphing that $\lim_{x \to \frac{1}{2}} \frac{\lvert 2x - 1 \rvert}{2x - 1}$ does not exist.
	2.6.3 Explain why $\lim_{x \to 3^+} \sqrt{9 - x^2}$ does not exist.
	2.6.4 Explain why $\lim_{x \to 3} [x]$ does not exist.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 3: Use the definition of continuity.

OBJECTIVES	SAMPLE MEASURES
3.1 Apply the definition of continuity and find excluded values of a discontinuous function.	3.1.1 Use the definition of continuity to show that $f(x) = \frac{x}{x^2 - 4}$ is continuous at $x = 5$.
	3.1.2 Find all values for which $f(x) = \frac{x^2 - 4}{x^2 - 7x + 6}$ is discontinuous. (a) 2, –2 (b) –3, –2 (c) –6, –1 (d) 6, 1
3.2 Use the statement: "If *f* is continuous on [*a*, *b*], then *f* has a maximum and a minimum on [*a*, *b*]."	3.2.1 Which of the following functions is not guaranteed to have a maximum or minimum on the given interval? (a) $y = \sin x$, $[-2, 2]$ (b) $y = \frac{1}{x}$, $[-2, 2]$ (c) $y = \ln(x)$, $[2, 4]$ (d) $y = \begin{cases} x + 3, & [0, 2] \\ x^2 + 1, & [2, 6] \end{cases}$
	3.2.2 Find the values of a and b so that $f(x)$ is continuous on $[-1, 3]$ and $f(x) = \begin{cases} x + 1, & \text{if } -1 \le x < 1 \\ ax + b, & \text{if } 1 \le x < 2 \\ -3x, & \text{if } 2 \le x \le 3 \end{cases}$

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 3: Use the definition of continuity.

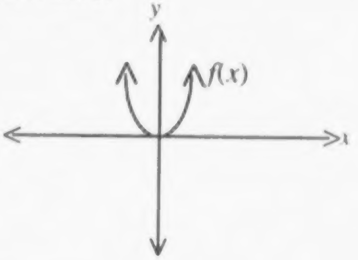
OBJECTIVES	SAMPLE MEASURES
3.3 Apply the intermediate value theorem.	3.3.1 If a function is continuous for all x and if f has a relative maximum at $(-1, 4)$ and a relative minimum at $(3, -2)$, which of the following statements must be true? (a) $f'(-1) = 0$ (b) The graph has a point of inflection somewhere between $x = -1$ and $x = 3$. (c) The graph of f has a horizontal tangent line at $x = 3$. (d) The graph of f intersects both axes.
	3.3.2 Show that $y = x^3 + 5x^2 + 3x - 2$ must have a zero between 0 and 1 without using an automatic grapher.
	3.3.3 Explain the intermediate value theorem in your own words.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: Use the concepts of differential calculus.

OBJECTIVES	SAMPLE MEASURES
4.1 Understand the concept of the derivative.	4.1.1 Given the graph below, sketch its derivative.  4.1.2 Use the definition of the derivative to determine whether $f'(2)$ exists for $f(x) = \lvert x + 2 \rvert$.
4.2 State and apply the definitions of the derivative.	4.2.1 Find $\lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$. (a) 1 (b) 0 (c) $f'(a)$ (d) does not exist 4.2.2 Use the definition of the derivative to find: $f'(x)$ for $f(x) = x^2 + 2x + 1$.
4.3 Find the derivatives of elementary functions.	4.3.1 Find the derivative of $f(x) = x^2$. (a) 2 (b) $2x$ (c) $\frac{1}{2}x$ (d) x 4.3.2 Find the derivative of $f(x) = ln\ (x)$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 4: Use the concepts of differential calculus.

OBJECTIVES	SAMPLE MEASURES
4.4 Find the derivative of a sum, product, and quotient.	4.4.1 If $f(x) = x^3 + 2x^2 + 5$, find $f'(x)$. (a) $3x + 2$ (b) $3x^2 + 5$ (c) $3x^2 + 4x + 5$ (d) $3x^2 + 4x$ 4.4.2 Find the derivative of $f(x) = e^x (x + 1)$. 4.4.3 Use the quotient rule to find the derivative of: $f(x) = \frac{7x - 3}{x^2 + 1}$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: Use the concepts of differential calculus.

OBJECTIVES	SAMPLE MEASURES
4.5 Find the derivative of a composite function (chain rule).	4.5.1 If $y = \sqrt{3-2x}$, find $D_x y$. (a) $\frac{1}{2\sqrt{3-2x}}$ (b) $-\frac{1}{\sqrt{3-2x}}$ (c) $-\frac{(3-2x)^{\frac{3}{2}}}{3}$ (d) $\frac{1}{3-2x}$
	4.5.2 If $f(x) = (8x^2 - 19x + 7)^5$, find $f'(0.2)$.
	4.5.3 If $y = \sin(ln(x))$, find $\frac{dy}{dx}$.
	4.5.4 If $f(2) = -4$ and $f'(2) = 3$, and $g(2) = 2$ and $g'(2) = 5$, find the derivative of $f(g(x))$ at $x = 2$.
4.6 Find the derivative of an implicitly defined function.	4.6.1 If $3x^2 + 2xy + y^2 = 2$, find the value of $\frac{dy}{dx}$ at $x = 1$. (a) 0 (b) 2 (c) 4 (d) not defined
	4.6.2 If $xy + \sin y = x^2$, find $\frac{dy}{dx}$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 4: Use the concepts of differential calculus.

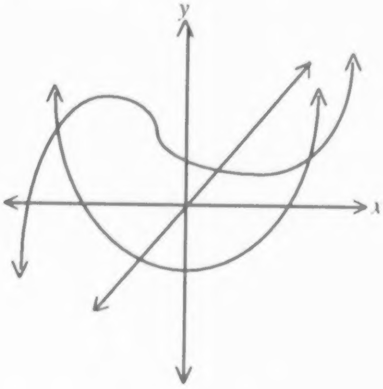
OBJECTIVES	SAMPLE MEASURES
4.7 Find the derivative of the inverse of a function.	4.7.1 If $f(x) = x^3 + x$ and h is the inverse of f, find $h'(2)$. (a) $\frac{1}{13}$ (b) $\frac{1}{4}$ (c) 1 (d) 4
	4.7.2 If $f(x) = \text{Arcsin}\sqrt{5x-1}$, find $f'(x)$.
4.8 Use logarithmic differentiation.	4.8.1 If $y = (x+1)^x$, find $\frac{dy}{dx}$. (a) $(x+1)^x\left(\frac{x}{x+1} + ln(x+1)\right)$ (b) $\frac{x}{x+1} + ln(x+1)$ (c) $x(x+1)^{x-1}$ (d) $(x+1)^{x-1}$
	4.8.2 If $y = x^x$, find $D_x y$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: Use the concepts of differential calculus.

OBJECTIVES	SAMPLE MEASURES
4.9 Find derivatives of higher order.	4.9.1 If $y = e^{nx}$, find $\frac{d^n x}{dx^n}$. (a) $n^n e^{nx}$ (b) $n!e^{nx}$ (c) ne^{ex} (d) $n^n e^x$ 4.9.2 If $f(x) = \cos 2x$, find $f''(x)$. 4.9.3 From the graph, identify $f(x)$, the first derivative, the second derivative, and discuss the fourth derivative. 

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 4: Use the concepts of differential calculus.

OBJECTIVES	SAMPLE MEASURES
4.10 Apply the Mean Value Theorem.	4.10.1 Why does the function $f(x) = x^{\frac{2}{3}}$ on $[-8, 8]$ not satisfy the conditions of the Mean Value Theorem? (a) $f(0)$ is not defined. (b) $f(x)$ is not continuous on $[-8, 8]$. (c) $f(x)$ is not defined for $x < 0$. (d) $f'(0)$ does not exist. 4.10.2 If $f(x) = \frac{2x+3}{3x-2}$ on $[1, 5]$, find all numbers that satisfy the Mean Value Theorem. 4.10.3 Let $f(x) = \cos x$. Find c that satisfies the Mean Value Theorem for f on $[2, 4]$. (a) 0.119 (b) 2.902 (c) 3.023 (d) 3.142

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 4: Use the concepts of differential calculus.

OBJECTIVES	SAMPLE MEASURES
4.11 Use the relation between differentiability and continuity.	4.11.1 If $y = f(x)$ is differentiable at a, then which of the following must be true? (a) $f(x)$ must be continuous at a (b) $f'(a) = 0$ (c) $f(x)$ is increasing (d) $(a, f(a))$ must be either a local maximum or a local minimum. 4.11.2 Which of the following statements is false? Give an illustration showing it is false. (1) Differentiability implies continuity. (2) Continuity implies differentiability. 4.11.3 Graph $y = x^{\frac{2}{3}} + \frac{1}{x+2}$ and discuss where the function is not differentiable and why.
4.12 Use L'Hopital's rule.	4.12.1 Find $\lim\limits_{h \to 0} \frac{\ln(e+h) - 1}{h}$. (a) 0 (b) $\frac{1}{e}$ (c) 1 (d) e 4.12.2 Explain why L'Hopital's rule does not apply to: $\lim\limits_{x \to 0} \frac{e^{x}}{x}$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 5: Apply the concepts of a derivative.

OBJECTIVES	SAMPLE MEASURES
5.1 Find the slope of a curve.	5.1.1 What is the slope of the curve $y^2 - xy - 3x = 1$ at the point $(0, -1)$? (a) -1 (b) -2 (c) 1 (d) 2
	5.1.2 Find the slope of $y = \tan^2(x)$ at $x = .44$.
5.2 Find the tangent line to a curve.	5.2.1 Write an equation for a tangent to the graph of $y = \text{Arcsin}\left(-\frac{x}{2}\right)$, where x and y are both equal to zero. (a) $x - 2y = 0$ (b) $x - y = 0$ (c) $x = 0$ (d) $y = 0$
	5.2.2 Find the tangent line to the curve $y^2 + xy = 3$ when $y = 1.5$. Illustrate this with a graph.
5.3 Find the normal line to a curve.	5.3.1 Find the slope of the normal line to the curve $y = \sin 2x + \cos x$ at $x = \frac{\pi}{6}$. (a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{3}{2}$ (d) -2
	5.3.2 Find the normal line to the curve $y = \frac{1}{x^2 + 1}$ when $x = 3.11$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: Apply the concepts of a derivative.

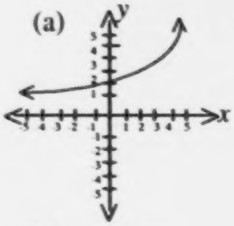
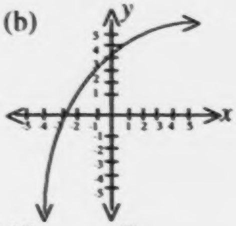
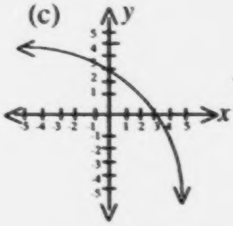
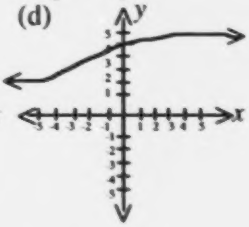
OBJECTIVES	SAMPLE MEASURES
5.4 Use the derivative to approximate values.	5.4.1 When small values of h are used, which of the following is the best approximation of $\sqrt[4]{16+h}$? (a) $4+\frac{h}{32}$ (b) $2+\frac{h}{32}$ (c) $4-\frac{h}{32}$ (d) $2-\frac{h}{32}$
	5.4.2 Find the approximate increase in the surface area of a spherical soap bubble if its radius increases from 3.0 to 3.25.
5.5 Use Newton's Method to approximate the zeros of a function.	5.5.1 Use Newton's Method to approximate $5\sqrt{3}$ with 3 iterations.
	5.5.2 Use Newton's Method to approximate the x coordinate of the point of intersection in the first quadrant of $y = x^2$ and $y = \sin x$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 5: Apply the concepts of a derivative.

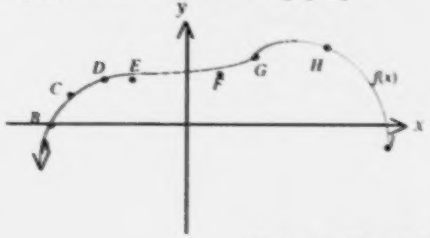
OBJECTIVES	SAMPLE MEASURES
5.6 Determine where a function is increasing and where it is decreasing.	5.6.1 If $y = f(x)$ and $y' < 0$ for all x, which could be a graph of $y = f(x)$? (a)  (b)  (c)  (d)  5.6.2 If $f(x) = 4x^3 - 3x^4$, determine where the curve is increasing and where the curve is decreasing. 5.6.3 Sketch a curve that is increasing on the interval (–3, 3) but decreasing everywhere else.
5.7 Find critical points, relative (local) and absolute maximum and minimum points.	5.7.1 For what value of k will $x + \frac{k}{x}$ have a relative maximum at $x = -2$? (a) –4 (b) –2 (c) 2 (d) 4 5.7.2 Find the critical numbers and the absolute maximum and minimum points of $f(x) = \sin 3x + 2 \cos 3x$, $0 \leq x \leq 2\pi$. 5.7.3 Explain why $f(x) = x^3 + 1$ has no local extrema.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: Apply the concepts of a derivative.

OBJECTIVES	SAMPLE MEASURES
5.8 Determine the concavity and points of inflection of a function.	5.8.1 If $f(x) = 3x^5 - 20x^3$, find all values for which the graph is concave up. (a) $x > 0$ (b) $-\sqrt{2} < x < 0$ (c) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$ (d) $x > \sqrt{2}$ 5.8.2 If $f(x) = xe^x$, determine where the curve is concave up and where it is concave down. 5.8.3 Where are the inflection points of $f(x) = 5x^4 - x^5$? (a) (0, 0) only (b) (3, 162) only (c) (4, 256) only (d) (0, 0) and (3, 162) 5.8.4 If $f(x) = (x + 2)(x - 2)^3$, find all points of inflection. 5.8.5 Identify the points of inflection of $f(x)$ from the following graph: 

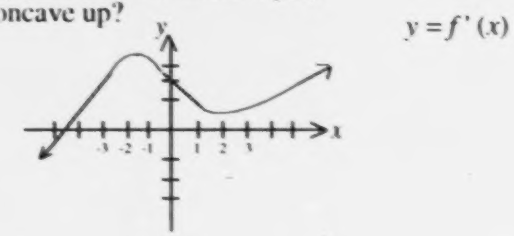
MATHEMATICS

Advanced Placement Calculus

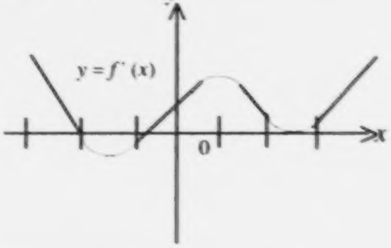
GRADE LEVEL: 9–12

COMPETENCY GOAL 5: Apply the concepts of a derivative.

<table>
<tr><th>OBJECTIVES</th><th>SAMPLE MEASURES</th></tr>
<tr><td>5.9 Use the graph of the derivative of a function to identify information about the function.</td><td>5.9.1 Given the graph of the derivative $f'(x)$ as follows, where is $f(x)$ concave up?



5.9.2 Use the graph of $f'(x)$ given and the fact that $f(-3) = 0$ to sketch a possible graph of f.


</td></tr>
<tr><td>5.10 Solve extreme value problems.</td><td>5.10.1 The sum of the squares of two positive numbers is 128. Find the minimum value of the product of the two numbers.

(a) 8 (b) 64 (c) 200

(d) 960 (e) none of these

5.10.2 A business finds that on the interval $t = [0, 12]$, where t is time in months, its profit is given by the function

$$P(t) = t^3 - 26t^2 + 148t.$$

At what time is its profit a maximum?</td></tr>
</table>

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 5: Apply the concepts of a derivative.

OBJECTIVES	SAMPLE MEASURES
5.11 Find the velocity and the acceleration of a particle moving along a line.	5.11.1 A particle moves along a horizontal line according to the law, $s = t^4 - 6t^3 + 12t^2 + 3.$ When is the velocity, v, increasing? (a) $t > 1$ (b) $1 < t < 2$ (c) $t < 2$ (d) $t < 1$ or $t > 2$
	5.11.2 The position function s of a point moving rectilinearly is given by $s(t) = 2t^3 - 15t^2 + 48t - 10$ where t is in seconds and $s(t)$ is in meters. (a) Find the acceleration when the velocity is 12m/sec. (b) Find the acceleration when V or $V(t) = 0$.
5.12 Find average rates of change.	5.12.1 If a particle moves along a horizontal line according to the law $x(t) = t^3 - 3t^2 + 2t - 1,$ what is the average velocity from $t = 1$ to $t = 2$? (a) 1 (b) 3 (c) 0 (d) –1
	5.12.2 If $s = 2t^3 + 3t - 1$, find the average velocity from $t = 1$ to $t = 3$.
	5.12.3 If $f(x) = 3\,ln(x)$, find the average rate of change of f on the interval $[\frac{1}{2}, 4]$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 5: Apply the concepts of a derivative.

OBJECTIVES		SAMPLE MEASURES	
5.13	Find instantaneous rates of change.	5.13.1	If a particle moves along a line according to the law $s = t^5 + 5t^4$, how often does it change direction? (a) 0 (b) 1 (c) 2 (d) 3
		5.13.2	If a certain substance is heated, the Celsius temperature after t minutes, $0 \le t \le 5$, is given by $g(t) = 30t + 6\sqrt{t} + 8$, find the instantaneous rate of change of $g(t)$ at $t = 4$.
5.14	Determine related rates of change.	5.14.1	A man six feet tall walks at the rate of 5 ft/s toward a streetlight that is 16 ft above the ground. At what rate is the tip of his shadow moving?
		5.14.2	Sand falls from a conveyor belt onto a conical pile at the rate of 10 ft^3/min. The radius of the base of the pile is always equal to one half its altitude. How fast is the altitude of the pile increasing when the pile is five feet high?

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: Use the concepts of integral calculus.

OBJECTIVES	SAMPLE MEASURES
6.1 Find antiderivatives.	6.1.1 Find an antiderivative of $3x^2 - 2x + 3$. (a) $x^3 - x^2 + 5$ (b) $3x^3 - x^2 + 3x + 5$ (c) $x^3 - x^2 + 3x + 5$ (d) $\frac{1}{2}(3x^2 - 2x + 3)^2 + 5$
	6.1.2 Find the most general antiderivative of $f(x) = 5x^{\frac{3}{2}} + 14x^{-2} - 7x + 13$.
6.2 Find the distance and velocity from acceleration with initial conditions.	6.2.1 The acceleration of a particle moving on a straight line is given by $a = \cos t$ and when $t = 0$ the particle is at rest. Find the distance it covers from $t = 0$ to $t = 2$. (a) sin 2 (b) 1 – cos 2 (c) cos 2 (d) sin 2 – 1
	6.2.2 Find an equation for velocity when $a = 7.5t^2 - 1.5t + 2$ and $v(1) = -2$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 6: Use the concepts of integral calculus.

OBJECTIVES	SAMPLE MEASURES
6.3 Solve simple (variable separable) first order differential equations.	6.3.1 Solve the differential equation $$\frac{dy}{dx} = \frac{1}{x} + 2x.$$ 6.3.2 Solve the differential equation $$\frac{dy}{dt} = 6y.$$ 6.3.3 Solve: If $\frac{dy}{dx} = y \sec^2 x$ and $y = 5$ when $x = 0$, then $y = ?$ (a) $e^{\tan x} + 4$ (b) $e^{\tan x} + 5$ (c) $5e^{\tan x}$ (d) $\tan x + 5$ (e) $\tan x + 5e^{x}$

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: Use the concepts of integral calculus.

OBJECTIVES	SAMPLE MEASURES
6.4 Apply solutions of $y' = ky$ to growth and decay problems.	6.4.1 The rate of decay of a radioactive substance is proportional to the amount of substance present at any time t. In 1840 there were 50 grams of the substance and in 1910 there were 35 grams. To the nearest gram, how many grams of the substance remained in 1990? (a) 18 (b) 20 (c) 23 (d) 107
	6.4.2 The number of bacteria in a culture increases from 300 to 900 in two hours. Assuming that the exponential law of growth holds, find a formula for the number of bacteria at any time t.
	6.4.3 Joe pours a cup of coffee that has a temperature of 95 degrees C. It cools off 5 degrees in 5 minutes. The temperature in the room is 20 degrees C. Find and graph a function to model this situation.
6.5 Use basic integration formulas.	6.5.1 Find $\int (3x^2 - 5x + 7)\,dx$.
	6.5.2 Find $\int (\sin x + e^x + \frac{1}{x})$ dx.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 6: Use the concepts of integral calculus.

OBJECTIVES	SAMPLE MEASURES
6.6 Use substitution to integrate.	6.6.1 Find $\int \sqrt{4-2t}\, dt$. (a) $-\frac{1}{3}(4-2t)^{\frac{3}{2}}+C$ (b) $\frac{2}{3}(4-2t)^{\frac{3}{2}}+C$ (c) $\frac{4}{3}(4-2t)^{\frac{3}{2}}+C$ (d) $-(4-2t)^{-\frac{1}{2}}$ 6.6.2 Find $\int t\sqrt{t+1}\, dt$. (a) $\frac{2}{3}(t+1)^{\frac{3}{2}}+C$ (b) $\frac{2}{15}(3t-2)(t+1)^{\frac{3}{2}}+C$ (c) $2t(t+1)+C$ (d) none of these

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: Use the concepts of integral calculus.

OBJECTIVES	SAMPLE MEASURES
6.7 Do simple integration by parts.	6.7.1 Find $\int \ln x \, dx$. (a) $\frac{1}{2}\ln^2 x + C$ (b) $x \ln x + x + C$ (c) $\frac{1}{2}\ln x^2 + C$ (d) $x \ln x - x + C$ 6.7.2 Find $\int xe^{2x} dx$.
6.8 Approximate the area under a curve using rectangles or trapezoids.	6.8.1 Let $y = x^2$. Approximate the first quadrant area under the curve from $x = 1$ to $x = 2$ using circumscribed rectangles with divisions at $x = \frac{4}{3}$ and $x = \frac{5}{3}$. (a) $\frac{7}{3}$ (b) $\frac{77}{27}$ (c) $\frac{50}{27}$ (d) $\frac{127}{54}$ 6.8.2 Approximate the first quadrant area under the curve $y = \frac{1}{x}$ from $x = 1$ to $x = 2$ (a) using inscribed rectangles with eight divisions. (b) using trapezoids with eight divisions. Discuss your results.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 6: Use the concepts of integral calculus.

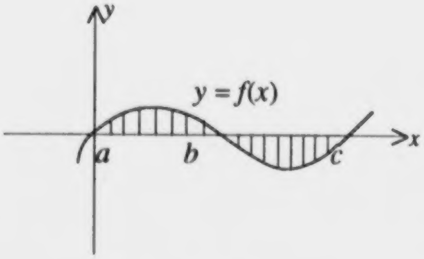
OBJECTIVES	SAMPLE MEASURES
6.9 Recognize the definition of the definite integral as the limit of a sum.	6.9.1 If $f(x)$ is continuous where $a \le x \le b$, and if this interval is partitioned into m equal subintervals of length Δx, and if x_k is a number in the kth subinterval, then find $\lim_{m \to \infty} \sum_{1}^{m} f(x_k)\, \Delta x.$ (a) $f(b) - f(a)$ (b) $\int_{a}^{b} f(x) dx$ (c) $F(a) - F(b)$ (d) none of these 6.9.2 Write as a definite integral $\lim_{m \to \infty} \sum_{1}^{m} f(x_k)\Delta x$, where $f(x) = x^3 - 1$ and $1 \le x \le 3$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: Use the concepts of integral calculus.

OBJECTIVES	SAMPLE MEASURES
6.10 Understand the concept of the definite integral.	6.10.1 Which of the following best represents the area of the shaded region?  (a) $\int_a^c f(x)dx$ (b) $\int_a^b f(x)dx + \int_b^c f(x)dx$ (c) $\int_b^c f(x)dx - \int_a^b f(x)dx$ (d) $\int_a^b f(x)dx - \int_b^c f(x)dx$

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 6: Use the concepts of integral calculus.

OBJECTIVES	SAMPLE MEASURES
6.10 Understand the concept of the definite integral.	6.10.2 Which of the following definite integrals is not equal to 0? (a) $\int_{-\pi}^{\pi} \sin^3 x \, dx$ (b) $\int_{-\pi}^{\pi} x^2 \sin x \, dx$ (c) $\int_{0}^{\pi} \cos x \, dx$ (d) $\int_{-\pi}^{\pi} \cos^3 x \, dx$ (e) $\int_{-\pi}^{\pi} \cos^2 x \, dx$

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 6: Use the concepts of integral calculus.

OBJECTIVES	SAMPLE MEASURES
6.11 Apply properties of the definite integral.	6.11.1 Given $\int_{1}^{10} f(x)dx = 8$ $\int_{1}^{5} f(x)\,dx = 6$ Find (a) $\int_{5}^{10} f(x)dx$ (b) $\int_{10}^{1} f(x)dx$ (c) $\int_{1}^{5} 7\,f(x)dx$ (d) $\int_{1}^{10} \left(f(x)+2\right)dx$ (e) $\int_{12}^{12} f(x)dx$ 6.11.2 Write as a single integral: $\int_{1}^{2} x^2\,dx + \int_{2}^{1} x\,dx.$

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 6: Use the concepts of integral calculus.

OBJECTIVES	SAMPLE MEASURES
6.12 Use the Fundamental Theorems $\frac{d}{dx}\int_a^x f(t)dt = f(x)$ and $\int_a^b f(x)dx = F(b) - F(a)$ where $F'(x) = f(x)$.	6.12.1 Find $\frac{d}{dx}\int_1^x \sqrt{t^4+1}\,dt$ 6.12.2 Let $F(x) = \int_0^{x^2} \sqrt{\sin t}\,dt$ of the following which best approximates $F'(1)$. (a) 0.0000 (b) 1.132 (c) 1.264 (d) 0.917 (e) 1.835 6.12.3 If F and f are continuous functions such that $F'(x) = f(x)$ for all x, then what is: $\int_a^b f(x)\,dx$? (a) $F'(a) - F'(b)$ (b) $F'(b) - F'(a)$ (c) $F(a) - F(b)$ (d) $F(b) - F(a)$ 6.12.4 $\int_1^2 \sqrt{5x}\,dx$ is approximately: (a) 2.981 (b) 3.354 (c) 13.628 (d) 20.442 (e) 2.726

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: Apply the concept of the definite integral.

OBJECTIVES	SAMPLE MEASURES
7.1 Find the average (mean) value of a function on an interval.	7.1.1 What is the average value of $\sqrt{x}$ over the interval $0 \leq x \leq 2$? (a) $\frac{1}{3}\sqrt{2}$ (b) $\frac{1}{2}\sqrt{2}$ (c) $\frac{2}{3}\sqrt{2}$ (d) $\frac{4}{3}\sqrt{2}$
	7.1.2 Find the average value of $f(x) = x^2 + 3x - 1$ on $[-1, 2]$.
	7.1.3 What is the average value of $f(x) = x \sin x^2$ on the interval $[0, 3]$?

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 7: Apply the concept of the definite integral.

OBJECTIVES	SAMPLE MEASURES
7.2 Find the area between curves.	7.2.1 What is the area of the enclosed region bounded by the curve $y = e^{2x}$ and the lines $x = 1$ and $y = 1$? (a) $\frac{2 - e^2}{2}$ (b) $\frac{e^2 - 3}{2}$ (c) $\frac{3 - e^2}{2}$ (d) $\frac{e^2 - 2}{2}$ 7.2.2 Find the area of the region bounded by the graphs of $y - x = 6$, $y - x^3 = 0$ and $2y + x = 0$. 7.2.3 The region R is enclosed by the graphs of $y = \frac{x}{x^2 - 2}$, $y = 0$, $x = 2$, and $x = k$, where $k > 2$. If the area of R is 1 square unit, then the approximate value of k is: (a) 2.727 (b) 4.096 (c) 10.450 (d) 14.213 (e) 22.256

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: Apply the concept of the definite integral.

OBJECTIVES	SAMPLE MEASURES
7.3 Find the volume of a solid of revolution about the axes or lines parallel to the axes.	7.3.1 The region in the first quadrant bounded by the graphs of $y = \sec x,\ x = \frac{\pi}{4}$ and the axes is rotated about the x–axis. What is the volume of the solid generated? (a) $\frac{\pi^2}{4}$ (b) $\pi - 1$ (c) π (d) 2π 7.3.2 Let *R* be the region in the first quadrant enclosed by the coordinate axes and the graphs of $y = 7 - 2x$ and $x = \sqrt{3}$. Of the following, which best approximates the volume of the solid generated when *R* is revolved around the x–axis? (a) 23.1 (b) 32.6 (c) 49.8 (d) 156.4

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 7: Apply the concept of the definite integral.

OBJECTIVES	SAMPLE MEASURES
7.3 Find the volume of a solid of revolution about the axes or lines parallel to the axes.	7.3.3 Set up but do not evaluate a definite integral that will find the volume of the solid generated by revolving about the x-axis the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$ for $0 \leq x \leq 1$. (a) $\pi \int_0^1 (2x - 2x^2)^2 \, dx$ (b) $\pi \int_0^1 (4x^2 - 4x^4) \, dx$ (c) $2\pi \int_0^1 x(2x - 2x^2) \, dx$ (d) $\pi \int_0^2 \left(\frac{y}{2} - \frac{y^2}{2}\right) dy$

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9-12

COMPETENCY GOAL 7: Apply the concept of the definite integral.

OBJECTIVES	SAMPLE MEASURES
7.3 Find the volume of a solid of revolution about the axes or lines parallel to the axes.	7.3.4 The region bounded by the lines $y = x^2$ and $y = 4$ is rotated about the line $x = 2$. Set up but do not evaluate a definite integral that will find the volume of the solid generated. (a) $2\pi \int_{-2}^{2} (2 - x)(4 - x^2)\,dx$ (b) $4\pi \int_{0}^{2} (2 - x)(4 - x^2)\,dx$ (c) $4\pi \int_{0}^{4} \sqrt{y}\,dy$ (d) $8\pi \int_{0}^{4} y\,dy$ 7.3.5 Find the volume of a solid generated by revolving about the line $y = 3$ the region bounded by the graphs of $y = x - 2$ and $x = y^2$.

MATHEMATICS

Advanced Placement Calculus

GRADE LEVEL: 9–12

COMPETENCY GOAL 7: Apply the concept of the definite integral.

OBJECTIVES	SAMPLE MEASURES
7.4 Find the volume of a solid with a known cross section.	7.4.1 The base of a solid is the circle $x^2 + y^2 = 9$. Every cross section of the solid by a plane perpendicular to the *x*-axis is a square with one edge in the base of the solid. Find the volume of the solid.
	7.4.2 Every cross section of a certain solid formed by planes perpendicular to the *y*-axis is a circle with one end of the diameter on $x = y^2$ and the other end on $x = 4 - y^2$. If the solid lies between the points of intersection of the curves, find the volume of the solid.